جامعة سيدي محمد بن عبد الله بفاس
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UNIVERSITÉ SIDI MOHAMED BEN ABDELLAH DE FES
الكلية متعددة التخصصات تازة
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FACULTÉ POLYDISCIPLINAIRE DE TAZA

The 5th edition of International Conference on Applied Mathematics

ICAM'2024

December 19-20, 2024
Polydisciplinary Faculty of Taza
Sidi Mohamed Ben Abdellah University
Taza-Morocco

Coordinators: SANHAJI Ahmed and HAMMOUMI Mohamed

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https://icam2024.sciencesconf.org

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- HAMMOUMI Mohamed, FPT, Sidi Mohamed Ben Abdellah University, Morocco

Keynote Speakers :

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- NITAJ Abderrahmane, University of Caen, France
- OUKNINE Youssef, Cadi Ayyad University, Morocco
- TOUHAFI Abdellah, Vrije Universiteit Brussel , Belgium
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FOREWORD AND ACKNOWLEDGEMENTS

The fifth edition of the International Conference on Applied Mathematics (ICAM'2024) is aimed to bring together people from all over the world working in the field of theoretical and applied mathematics and related fields to exchange ideas, to initiate future collaborations and interact with each other.

This conference has several major objectives, in particular :

- To bring the doctoral researchers to exchange ideas with scientific specialists in their respective fields of study and seek to discover.
- To consolidate the scientific cooperation between the university and the socio-economic environment in the field of applied sciences.
- To allow young researchers to present and discuss their research work before a panel of specialists and university professors.

The conference which took place in Taza-Morocco, from December 19 to December 20, 2024, featuring 5 keynote speakers and several oral talks, aims to gather experts in theoretical and applied mathematics.

We wish to acknowledge the conference program committee and reviewers, for their substantial contributions and our institutions, for their support. Also, we wish you all a pleasant and memorable conference with open-minded and fruitful discussions
Sincerely,

On behalf of Organizing Committee of ICAM'2024
Profs. Ahmed SANHAJI and Mohamed HAMMOUMI
Laboratory of Engineering Sciences (LSI)
Polydisciplinary Faculty of Taza
Sidi Mohamed Ben Abdellah University, Fez, Morocco

Detailed program

5TH EDITION OF INTERNATIONAL CONFERENCE ON APPLIED MATHEMATICS (ICAM'2024)

December 19-20, 2024

Polydisciplinary Faculty of Taza (FPT)

Taza-Morocco

Planning

Thursday 19 December 2024		
Time	**Conference room/Salle de conférence**	
09:00-10:00	Registration and welcome	
10:00-10:45	Opening Ceremony	
10:45-11:45	Speaker 2: Pr. Abdessamad Tridane	Chair: A. El azzouzi
	Title: «New Optimal Control Approach to Contain Epidemic in the Case of Limited Resources»	
Coffee Break		
Session 1: Nonlinear Analysis and Applications 1		
Chair: A. Dakkak and M. D. Belghiti		Room : Salle de conférence
Speaker		**Title**
12:15-12:30	Mohamed ELMASSAOUDI	On the Characterization of the Spectrum of a Fourth-Order Eigenvalue Problem
12:30-12:45	Mohammed BARGHOUTHE	Existence of solutions for a discrete problem with mixed boundary conditions
12:45-13:00	Mohamed TIAL	Indépendance et stabilité de quelques équations fonctionnelles
13:00-13:15	Fatna JAAFRI	Existence and non existence of solutions for a bi-nonlocal p(x)-Kirchhoff equation
13:15-13:30	Abdellah EL ALLAOUI	Common properties of the bounded right linear operators *AB* and *BA* in the quaternionic setting
13:30-13:45	Ayoub OUHABI	System of functional equations on semigroups
Session 2: Partial Differential Equations 1		
Chair: S. Lalaoui Rhali and M. Mekkour		Room: B1
Speaker		**Title**
12:15-12:30	Ahmed ABERQI	Double Phases Problems: Insight and new trends
12:30-12:45	Mohammed El ANSARI	Non-coercive differential inclusions problems in anisotropic Sobolev spaces with variable exponent and L^1-data
12:45-13:00	Rachid El Ayadi	Internal and gradient stabilization of an unbounded bilinear systems: parabolic and hyperbolic cases
13:00-13:15	Rachid ELHARCH	Existence of a solution for nonlinear degenerate elliptic unilateral problems
13:15-13:30	Ouidad AZRAIBI	On some nonlinear elliptic equations with measurable boundary conditions in anisotropic weighted Sobolev spaces
13:30-13:45	Mouad ALLALOU	On a class of $p(z)$-biharmonic Kirchhoff type problems with indefinite weight and no-flow boundary condition

Session 3: Stochastic Analysis and Probability Theory

Chair: M. Nemiche and M.M. CHEMS-EDDIN		Room: B4
Speaker		**Title**
12:15-12:30	Jaouad BOURABIAA	Stochastic Aggregation-Diffusion Equation: Analysis via Dirichlet Forms
12:30-12:45	Soukaina BIGANZINE	Global Existence of Solutions of Stochastic Aggregation Equations with Random Diffusion
12:45-13:00	Mohamed BOURHAIL	Accurate Closed-Form Approximations for the Sum of a Ratio of Uncorrelated Rayleigh Random Variables
13:00-13:15	Fatima Zahra ASSILA	Exploring Complex Monge-Ampère Measures : Insights from Projective Logarithmic Potentials
13:15-13:30	Mohamed EL MOUSTAFID	Convergence of martingale: Application to uniform amart
13:30-13:45	Abdenbi EL AZRI	Simulated annealing estimation for three parameters in a generalized stochastic Raleigh diffusion process
13:45-14:00	Anouar BOUTAHYRY	Convergence and decomposition theorems of set-valued mils

Session 4: Control Theory 1

Chair: Z. Rahou and M. Elmassoudi		Room: C4
Speaker		**Title**
12:15-12:30	Mohamed ERRACHIDI	Contraction-Based Hidden State Regularization in Convolutional Neural Networks
12:30-12:45	Wadii GHANDOR	The controllability of fractional differential system with state and control delays
12:45-13:00	Hamza EL BOUTAHIRI	Strong Stabilization of an Unbounded Bilinear System with Time Delay
13:00-13:15	Oussama EJJARJARI	feedback strong stabilization for a semilinear control system of time-dependent delay
13:15-13:30	Mohammed ZARHOUNI	Pseudo almost periodic solution of a delayed quaternion-Valued Fuzzy recurrent neural networks model on time scales
13:30-13:45	Mohssine ES-SAIYDY	Almost periodic solution of Lotka-Volterra neural networks.
13:45-14:00	Aissam ICHATOUHANE	Applying convexificators in nonsmooth multi-objective semi-infinite fractional interval-valued optimization

Session 5: Algebra 1

Chair: A. Boua and A. Soullami		Room: C5
Speaker		**Title**
12:15-12:30	Iz-iddine EL-FASSI	Approximate solution for a certain type of functional equations
12:30-12:45	Abdelilah ZERBANE	On the structure of quotient quasi-rings
12:45-13:00	Sara AHALLAL	Confluent hypergeometric functions in continued fractions
13:00-13:15	Safia BATLA	The First Cohomology Group of the Units of a Real Biquadratic Field and Its P´olya Group
13:15-13:30	Jawad BOUTARFASS	Stability of functional inequalities in quasi-Banach spaces
13:30-13:45	Imane EL KHIR	Commutative rings in which every ideal is S-2-absorbing

Session 6: Cryptography and Coding Theory 1

Chair: A. Ahaitouf and A. Chillali		Room: Salle de décanat
Speaker		**Title**
12:15-12:30	Mohammed SAHMOUDI	On monogeneity of nonic relative number field
12:30-12:45	Mohamed EL BACHRAOUI	N-tuple sum analogues for Ramanujan-type congruences
12:45-13:00	Noureddine ESSAIDI	Advancing the Construction of Quantum Codes over Non-Chain Rings: A Generalized Approach
13:00-13:15	Hafsa EL KHATTABI	Characterizing Subgroups of Class Groups in Biquadratic Fields
13:15-13:30	Bilal MEJMAA	A new SRTOMS algorithm for 5G New-Radio Standards
13:30-13:45	Youness TALEB	Error Correction in IoT LoRa Systems: Polar Codes for Enhanced Communication Reliability
13:45-14:00	Youness ERRADI	p^n -Power relatives extensions with power integral basis

Lunch

Time	Conference room/Salle de conférence	

15:30-16:30	**Speaker 3 : Pr. Abdellah TOUHAFI**	**Chair: K. El moutaouakil**
	Title: «Mathematical Tools and Techniques for Cough Analysis Using Acoustic Camera Technology»	

Session 7: Nonlinear Analysis and Applications 2

Chair: K. Belhadj and M. Tial		**Room : Salle de conférence**
Speaker		**Title**
16:30-16:45	**Hajar EL BALLOUT**	**On the problem of unique continuation for a nonlinear elliptic equation**
16:45-17:00	**Anass LAMAIZI**	**Existence of solutions for p-Laplacian parabolic equation**
17:00-17:15	**Abdelmajid BOUKHSAS**	**Existence of solutions for a Steklov eigenvalue problem**
17:30-17:45	**Mahmoud EL AHMADI**	**Multiplicity results for Kirchhoff-double phase problem with nonlinear boundary condition**
17:45-18:00	**Ibrahim Chamlal**	**Etude de la multiplicit2 des solutions faibles d'un problème singulier ($p(x)$, $q(x)$)-biharmonique avec conditions de non-flux**
18:00-18:15	**Mohamed BOUABDALLAH**	**Multiple homoclinic solutions for a class of discrete fractional p-Laplacian equations**

Session 8: Control Theory 2

Chair: A. Essahlaoui and A. Aberqi		**Room: B2**
Speaker		**Title**
16:30-16:45	**Marwan FAKHRI**	**Strong stabilization and decay estimate for a class of semilinear time-delay systems of neutral type**
16:45-17:00	**Lamiae SEDDATI**	**Neural Network Controllability of financial Dynamic Systems**
17:00-17:15	**Jawad SALHI**	**Indirect controllability of coupled degenerate Euler-Bernoulli beam equations**
17:30-17:45	**Hamid BOUTANFIT**	**Viscosity solutions of HJB equation for a model of wastewater treatment**
17:45-18:00	**Mohammed ERRAKI**	**Optimal feedback control stabilisation for a class of bilinear delayed systems of neutral type**
18:00-18:15	**Chaimae OUCHICHA**	**A novel fuzzy clustering approach for robust MRI brain segmentation**
18:15-18:30	**Mohamed OHDA**	**On the partial calmness condition for an interval-valued bilevel optimization problem**

Session 9: Partial Differential Equations 2

Chair: C. Yazough and A. Salmani		**Room: C8**
Speaker		**Title**
16:30-16:45	**Safae El Alaoui**	**Bilinear stabilization of second order evolution equations by a class of bilinear unbounded feedbacks**
16:45-17:00	**Mohamed KNIFDA**	**On a class of double phase problem involving potentials terms**
17:00-17:15	**Khalid SOUALHINE**	**On a generalized $p(x)$-biharmonic problem with two nonlocal terms**
17:30-17:45	**Zoubida Echchaffani**	**Existence of mild solution and approximate controllability for nonlinear fractional neutral evolution systems**
17:45-18:00	**Abdellah HAMIDI**	**The Nehari manifold for anisotropic Kirchhoff problems involving variable singular exponent and critical terms**
18:00-18:15	**Mohammed EL FATRY**	**A nonlinear elliptic problems with natural growth and integrable data**

Session 10: Cryptography and Coding Theory 2

Chair : A. Mouhib and M. Abarkan		**Room: C12**
Speaker		**Title**
16:30-16:45	**Aziz BOULBOT**	**Elliptic curves over the ring $Fq[e]$, $e^3 = e$**
16:45-17:00	**Jalal DIDI**	**On monogenity of certain septic number fields defined by $x^7 + ax^6 + b$**
17:00-17:15	**Hussain BEN-AZZA**	**Codes on Dessins d'Enfants**
17:30-17:45	**Abdelali GRINI**	**Twisted Hessian Curve ElGamal Cryptosystem**
17:45-18:00	**Ali ECH-CHAKOURI**	**Some Characterizations of Codisk-Cyclicity for Linear Relations**
18:00-18:15	**Fatima HMIDANI**	**Techniques in image encryption and decryption**
18:30-18:45	**Abdelghani ASSARRAR**	**On greenberg's conjectures**

Session 11: Algebra 2

Chair: M. Taous and M. Ouriagli		**Room: C13**
Speaker		**Title**

16:30-16:45	Ayoub OUAZZA	Some identities arising homoderivations in prime rings and Banach algebras
16:45-17:00	Abderazak SOULLAMI	Algebraic number fields generated by an infinite family of monogenic trinomials
17:00-17:15	Fedoua SGHIOUER	Rational approximation of quasi-arithmetic power and heron operator means
17:30-17:45	Mouhssine EL-ARABI	On proper strong property (A) for rings and modules
17:45-18:00	Imad BOUROUAHA	An Algebraic Perspective on Generalized Fredholm Operators via Projection Techniques
Session 12: Artificial intelligence 1		
Chair: K. El Moutaouakil and A. Hallaoui		**Room: Salle de décanat**
Speaker		**Title**
16:30-16:45	Omar BAHOU	Fuzzy-PID controller
16:45-17:00	Khadija MOUATASSIM	An Innovative Intrusion Detection Model for VANETs Using Convolutional Neural Networks
17:00-17:15	Chaimae OUCHICHA	A novel fuzzy clustering approach for robust MRI brain segmentation
17:30-17:45	Azedine OUHMID	Investigating the Memory Effect in Supply Chain Management Using Fractional Calculus Models
17:45-18:00	Ayoub BENACHOUR	Bridging AI and Culture: Developing an Autonomous Robot with Darija Conversational Capabilities and Object Recognition
18:00-18:15	Ali BOUFSSASSE	An Innovative Dropout Strategy for Enhancing Robustness in GNNs
18:15-18:30	Safaa SAFOUAN	Improved Clustering with Fractional Fuzzy C-Means: via Long-Memory Effects and Geometric Indices
18:30-18:45	Nassira MADANI	An Innovative Approach Utilizing Artificial Neural Networks and the Hybrid GA-IPA Technique to Solve Fractional Non-linear Dynamical System
18:45-19 :00	Mohammed ZIANE	Resilient Strategies for Complex Markets: The Role of AI, Robust Optimization, and Dynamic Approaches in Portfolio Management

Friday 20 December 2024		
Time	**Conference room/Salle de conférence**	
09:00-10:00	**Speaker 5: Pr. Abderrahmane NITAJ**	**Chair: S. Lalaoui Rhali**
	Title: « Lattice Based Post Quantum Cryptography »	
10:00-11:00	**Speaker 6: Pr. Ali KACHA**	**Chair: A. Boua**
	Title: « Transcendence of some continued fractions »	
Coffee Break		
Session 13: Partial Differential Equations 3		
Chair: M. C. Hassib and D. Achemlal		**Room : C4**
Speaker		**Title**
11:30-11:45	Anouar MARSOU	Nonlinear Parabolic Problem with $L1$-data in Orlicz Spaces
11:45-12:00	Said AIT DADA ALLA	Study of Nonlinear Elliptic Equations with Measurable Boundary Conditions in Anisotropic Weighted Sobolev Spaces
12:00-12:15	Adnan LAMTARAH	Quasilinear degenerate elliptic unilateral problems with measure data in the Anisotropic Sobolev Space
12:15-12:30	Jalal EL HAJOUJI	Entropy solution for some parabolic problems nonlinear in Musielak spaces with $L1$ Data
12:30-12:45	Abdelhak EL AOULA	Stabilization for a class of unbounded bilinear control system in Banach space
12:30-12:45	Mohammed ABDELLAOUI	Renormalized solution for a fractional (s, p)-Laplacian parabolic problem with diffuse measure data
Session 14: Theory control 3		
Chair: S. El Hazzat and I. El-fassi		**Room: C12**
Speaker		**Title**
11:30-11:45	Mohammed BENGRICH	Stabilization and decay estimate for a class of neutral functional differential equations
11:45-12:00	Atmane EL HOUCH	Strong stabilization for a class of bilinear systems of neutral type with discrete multi-delays
12:00-12:15	Hamza BEN BRAHIM	Observability for a Class of Ψ-time -fractional linear systems
12:15-12:30	Asmaa AADI	On The Controllability of a class of Time-Fractional

		Systems with order between 1 and 2.
Session 15: Numerical Analysis and Scientific Computing 1		
Chair: A. Raji and Y. Benali		**Room: C6**
Speaker		**Title**
11:30-11:45	**Oumayma JAHID**	**The Finite Element Method in Anisotropic Sobolev Spaces**
11:45-12:00	**Omar RHOUNI**	**Numerical Methods for Solving the One-Dimensional Birkhoff Polynomial Interpolation Problem**
12:00-12:15	**Mohammed ZARHOUNI**	**pseudo almost periodic solution of a Delayed Quaternion-Valued Fuzzy Recurrent Neural Networks model on Time Scales**
12:15-12:30	**A.hmed EL-ALAOUI**	**Bernstein polynomials and local polynomial for the smooth estimation of regression**
Session 16: Cryptography and Coding Theory 3		
Chair : A. Kacha and M. Sahmoudi		**Room: Salle de conférence**
Speaker		**Title**
11:30-11:45	**Amine MARZOUKI**	**Dénombrement des points d'une courbe elliptique**
11:30-11:45	**Karim BOULAJHAF**	**Cyclicity of the 2-decomposed unramified Iwasawa module**
11:45-12:00	**Aziz JAARI**	**Investment of some results and properties of graphs in some algebraic structures**
12:00-12:15	**Azddine ADDADI**	**Information Geometry Approach for MIMO Communication and Capacity**
12:15-12:30	**Ahmed LAHLOU**	**Some Results on Elliptic Curves over Finite Fields**
12:30-12:45	**Zakariae CHEDDOUR**	**Double cyclic Codes over $\mathcal{R}$ = Z2 + ζZ2**
12:45-13:00	**Abdelghani LARHLID**	**On a problem of Pillai involving S-units and Perrin numbers**
Session 17: Dynamical Systems and Biomathematics 1		
Chair: A. El azzouzi and A. Labzour		**Room: C3**
Speaker		**Title**
11:30-11:45	**Ayoub CHEDDOUR**	**Stabilization of bilinear systems with distributed delays using the Banach state space decomposition method**
11:45-12:00	**Ibtissam BENAMARA**	**Stability analysis of a predator-prey eco-epidemic model with hunting cooperation and fear effects**
12:00-12:15	**Mohamed IGUERNANE**	**CNN-Based Facial Emotion Recognition: A Comprehensive Survey of Recent Advances and Challenges**
12:15-12:30	**Chaimaa ASSILA**	**Hopf bifurcation in a delayed fractional-order prey-predator model with Holling type II functional response, incorporating a refuge area for prey in the presence of toxicity**
12:30-12:45	**Aiman MDAGHRI**	**Distributions Stationnaires dans les Modèles Epidémiques Stochastiques : Une Approche Ergodiq**
12:45-13 :00	**Hicham MAADAN**	**Some Mathematical models based on industry 4.0 for healthcare**
13 :00-13 :15	**Abdellah OUAKKA**	**Modeling and Control of Chickenpox Transmission Using Age-Specific Vaccination and Immunity: An Analytical and Numerical Study**
	Lunch	
Time	**Conference room/Salle de Conférence**	
15:30-16:30	**Round table to discuss collaborations**	
16:30	**Closing ceremony**	

ABSTRACTS

ALGEBRA

5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

Algebraic number fields generated by an infinite family of monogenic trinomials

*** Soullami Abderazak[1] and Daniel C. Mayer[2]**

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Abstract:

For an infinite family of monogenic trinomials $P(X) = X^3 - 3rbX^2 - b \in \mathbb{Z}[X]$, arithmetical invariants of the simply real cubic number fields $L = \mathbb{Q}(\theta)$ with negative discriminants $d_L < 0$, generated by a zero θ of $P(X)$, and of their Galois closures $N = L(\sqrt{d_L})$, are determined. The conductor f of the cyclic cubic relative extension N/K, where $K = \mathbb{Q}(\sqrt{d_L})$ denotes the unique quadratic subfield of N, turns out to be of the form $3^e b$ with $e \in \{1, 2\}$, which admits statements concerning primitive ambiguous principal ideals in L and N, the chain of lattice minima $(\Theta_i)_{i\in\mathbb{Z}}$ in the maximal order $\mathcal{O}_L$ of L, its primitive period length $\ell(\mathcal{O}_L)$, and the number m of non-isomorphic cubic fields $L_1, \ldots, L_m$ sharing a common discriminant $d_{L_i} = d_L$.

Key Words: *Trinomials, algebraic number fields, Galois closures, equation orders, maximal orders, monogeneity, cubic fields, indices, discriminants, conductors, ambiguous principal ideals, capitulation kernels, fundamental systems of units, lattice minima, Voronoi algorithm, small period lengths, Hilbert class field*

References

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Some identities arising homoderivations in prime rings and Banach algebras

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Abstract:

Let $\mathcal{A}$ be a Banach algebra over $\mathbb{R}$ or $\mathbb{C}$ with the center $\mathcal{Z}(\mathcal{A})$. In this article, we show that, if a homoderivation of $\mathcal{A}$ satisfies some local differential identities, then $\mathcal{A}$ must be commutative. We give some examples and applications.

Key Words: *Banach algebra, contunuity, closed subspace, derivation, open subspace.*

Introduction: *In all that follows, will be an associative ring with center and usually, is 2-torsin free, if whenever $2x = 0$ with $x \in$ forces $x = 0$. The ring is said to be prime if for any $x, y \in$, $xy = \{0\}$ implies either $x = 0$ or $y = 0$. The Lie product and Jordan product of $x, y \in$ are denoted $[x, y]$ and $x \circ y$ respectively, where $[x, y] = xy - yx$ and $x \circ y = xy + yx$. An additive mapping $h :\to$ is called a homoderivation if $h(xy) = h(x)h(y) + h(x)y + xh(y)$ hold for all $x, y \in$ in [13]. An example of such mapping is to let $h(x) = u(x) - x$ for all $x \in$ where u is an endomorphism on . A linear mappind $d :\to$ is said to be a derivation if $d(xy) = d(x)y + xd(y)$ hold for all $x, y \in$ and is called an inner derivation if $d(x) = [a, x]$ for all $x \in$ and for some fixed element $a \in$. It is clear that a homoderivation is also a derivation if $h(x)h(y) = 0$ for all $x, y \in$. Let be a Banach algebra over the complex field and g be an automorphism of . A linear map $f :\to$ is called a linear skew-derivation if $f(xy) = f(x)y + g(x)f(x)$ for all $x, y \in_A$. When $g = I_A$ on a skew-derivation is simply an ordinary linear derivation.*

Finding combinations of properties that require a ring to be commutative is a classic ring theory issue. A number of ring theory theorems, primarily attributed to Herstein, aim to demonstrate that a given ring must be commutative as a result of the conditions that appear to be weak to show commutativity. Consider the following theorems of Herstein [[5], p412] which states that a ring is commutative if for each x and y in there is a positive integers $n > 1$ such that $x^n - x$ permutes with y.

The correlation between a ring's commutativity and the existence of specific types of derivation has attracted constant interest over the past several years. The crucial objective is to reduce the problem of purmetation of the elements of a ring under some differential identities to ensure commutativity. A stunning result was displayed by Posner in 1957 *states that: If a prime ring has a non zero derivation which is centralizing on the entire ring, then the ring must be commutative. Motivated by this result, a comparable result was attained for automorphisms by J. Mayne [10] this work has also been extended in various directions. Later, in [1] Bell and Daif started researching a certain type of map that preserves commutativity in the following way: "Let $\mathcal{S}$ be a subset of a ring . A map $f : \mathcal{S} \to$ is called strong commutativity preserving (SCP) on $\mathcal{S}$ if $[f(x), f(y)] = [x, y]$ for all $x, y \in \mathcal{S}$". More specifically, they proved that must be commutative if is a prime ring and admits derivation or non-identity endomorphism which is SCP on right ideal of .*

In Banach algebras, Yood [?] proved that if a semiprime Banach algebra having two nonvoid open subset $\mathcal{H}_1$ and $\mathcal{H}_2$ verify for all $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$ there are strictly positive integers n, m such that $[x^n, y^m] = 0$, then is commutative. Inspired by the work of Ali and khan [?] and Yood [?, ?, ?], Rehman [12] explore strong commutativity preserving skew-derivation on Banach algebras and proved that [[12],Theorem 3] if be a prime Banach algebra and f be a continuous linear skew derivition. Suppose thet there are non-empty open subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ of such that $[f(x^m), f(y^n)] - [x^m, y^n] \in$ for each $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$, then is commutative. In [[9],Theorem 1], M. Moumen, L. Taoufiq and L. Oukhtite shown that if be a prime Banach algebra and $\mathcal{H}_1$, $\mathcal{H}_2$ nonvoid open subsets of , d a continuous non-injective derivation of such that: for all $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$ there are a positive integers p, q such

that $d(x^p y^q) + [x^p, y^q] \in$, then must be commutative. After, M. Moumen, L. Taoufiq and A. Boua [8] proved that if be a prime Banach algebra and $\mathcal{H}_1$, $\mathcal{H}_2$ nonvoid open subsets of , d a continuous non-injective derivation of satisfying one of the following conditions:

1. *for all $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$, there exists $(p, q) \in \mathbb{N}^* \times \mathbb{N}^*$ such that $d(x^p y^q) \pm x^p \circ y^q \in$.*
2. *for all $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$, there exists $(p, q) \in \mathbb{N}^* \times \mathbb{N}^*$ such that $d(x^p \circ y^q) \pm [x^p, y^q] \in$,*

then is commutative (for more examples, see[9, 8]).
Motivated by these results, we find an alternative direction for commutativity for Banach algebras using homoderivation, this article's is going to focus on shown some results with similar conclusion, but with other identities with homoderivations. In particular, we have proved that if a prime Banach algebra has continuous linear homoderivation and there are two nonvoid open subsets $_1$, $_2$ of satisfying: for all $(x, y) \in_1 \times_2$ there are strictly positive integers p, q such that $h(x^p y^q) + x^p \circ y^q \in$, then must be commutative. In this context, other similar result have been found.

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When is the idealization $R \propto M$ an $\mathcal{A}$-ring?

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Abstract. We present an answer to a problem raised by D.D. Anderson and S. Chun in [3] on characterizing when the idealization $R \propto M$ of a ring R on an R-module M is an $\mathcal{A}$-ring (resp., an $\mathcal{SA}$-ring) in terms of module-theoretic properties of R and M. Also, we are concerned with an open question asked by these two authors which reads the following: What modules over a given ring R are homomorphic images of modules satisfying the strong Property $\mathcal{A}$?[3, Question 4.4 (1)]. This paper highly contributes to answer such a question.

Keywords: $\mathcal{A}$-module; annihilator; $\mathcal{A}$-ring; idealization; $\mathcal{SA}$-module; $\mathcal{SA}$-ring; zero divisor.

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Rational approximation of quasi-arithmetic power and heron operator means

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Abstract:

The aim of this paper is to provide an efficient method for computing the quasi-arithmetic power means and the Heron operator means of two positive definite matrices, such that we give an expansion of these means using rational approximations. At the end, we discuss several numerical examples that illustrate the theoretical results.

Key Words: *Rational approximation, positive definite matrix, quasi-arithmetic power means.*

Introduction: *The study of matrix means, such as quasi-arithmetic and Heron means, is crucial in matrix analysis and operator theory. These means provide valuable tools for interpolation and approximation, preserving essential properties such as positive definiteness and spectral structure. The quasi-arithmetic power mean, defined for positive definite matrices, generalizes classical means (arithmetic, geometric, harmonic) by introducing parameters for exponentiation and weighting, allowing the mean to adapt to various application contexts. Similarly, Heron means offer an interpolation framework between arithmetic and geometric means, facilitating applications in optimization and signal processing.*

Rational approximation of these means offers an efficient approach for numerical computations, allowing for high accuracy and spectral stability in both computational and theoretical applications. Rational approximations not only enhance computational speed but also ensure desirable spectral properties, making them essential in fields such as matrix theory and dynamical systems.

In this article, we propose an efficient method for calculating the quasi-arithmetic power means and Heron operator means for two positive definite matrices. We establish an expansion of these means using rational approximations, aiming to improve the accuracy and speed of the calculations. Finally, several numerical examples are presented to illustrate the theoretical results and demonstrate the effectiveness of our approach.

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An Algebraic Perspective on Generalized Fredholm Operators via Projection Techniques

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Abstract:

We will give an algebraic characterization of generalized Fredholm operators in terms of projections by means of [14, Therorem 1.1]. More precisely, for $T \in \mathcal{B}(\mathcal{X})$ which is the Banach algebra of all bounded linear operators on a Banach space $\mathcal{X}$, we shall prove the following fact: T is a generalized Fredholm operator if and only if there exists a projection P which commutes with T such that, TP is a Fredholm element in $P\mathcal{B}(\mathcal{X})P$ with jump $j_{P\mathcal{A}P}(TP) = 0$ and $(Id_{\mathcal{X}} - P)T$ is a nilpotent element in $soc((Id_{\mathcal{X}} - P)\mathcal{B}(\mathcal{X})(Id_{\mathcal{X}} - P))$. This characterization can open the way for us to extend the above Theorem to the more general context like that of a semisimple complex Banach algebras.

Key Words: Fredholm operators, Generalized Fredholm operators, Fredholm elements, Generalized Fredholm elements, Semisimple Banach Algebras, Socle, jump.

Introduction: According to Caradus [6] a bounded operator T on $\mathcal{X}$, which is a Banach space $\mathcal{X}$, is said to be generalized Fredholm if there exists a bounded operator S on $\mathcal{X}$ such that $TST = T$ and $I - ST - TS$ is a Fredholm operator on $\mathcal{X}$. Examples of generalized Fredholm operators are inversible, finite-dimensional and Fredholm operators and projections. This class of operators has been studied in several papers by Schmoeger [13, 14, 10]. In accordance with [1, Page 79], a remarkable result is that an operator T is generalized Fredholm if and only if there exist two closed invariant subspaces $\mathcal{X}_1$ and $\mathcal{X}_2$ such that $\mathcal{X} = \mathcal{X}_1 \bigoplus \mathcal{X}_2$, $T_{|\mathcal{X}_1}$ is Fredholm and $T_{|\mathcal{X}_2}$ is a finite rank nilpotent operator, see [14, Theorem 1.1]. Using this last result we will give in this paper an algebraic characterization of generalized Fredholm operators, in terms of the projections, in the horizon to extend it to the more general context like that of a semisimple complex Banach algebra. For reference, we have done this previously, in [9], with regard to the results found in the reference [13].
The present paper is originally motivated by the desire (driven by ambition) to generalize [14, Theorem 1.1] to the complex semisimple Banach algebras. Although we could not fulfill this desire in this work, we were able, in return, to formulate a conjecture related to this. Hence, the main theorems presented in this article can be considered as justifications for this conjecture.
In order to prove the main theorems we first need some ideas and results on this context; thus in Section 2, we will expose them in the form of lemmas. Finally, the last Section will be devoted to the proofs of the main results.

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Commutative rings in which every ideal is S-2-absorbing

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Abstract:

Let R be a commutative ring with identity and S be a multiplicative subset of R. In this paper, we introduce and study the notion of S-2-absorbing rings. We define a ring R to be an S-2-absorbing ring if every proper ideal of R disjoint with S is S-2-absorbing. Several properties and characterizations of S-2-absorbing rings and ideals are given. Moreover, we study the transfer of the above properties to some constructions of rings such as trivial ring extensions and amalgamation of rings along an ideal.

Key Words: 2-absorbing ideal, S-2-absorbing ideal, S-2-absorbing ring

Introduction: Throughought this paper, all rings considered are assumed to be commutative with identities and all modules are assumed to be unital. In [4], the authors introduced the concept of "almost finitely generated". Later, Anderson and Dumitrescu in [1] abstracted this notion to any commutative ring and introduced the concepts of S-Noetherian rings. This initiated a series of papers that investigate S-Noetherianity on a specific class of rings, and generalize several notions in multiplicative ideal theory to their S-versions. The concept of prime ideals has an important role in the theory of commutative rings. Several generalizations of prime ideals are studied in the literature. Among them, the notion of 2-absorbing ideals due to A. Badawi in [2]. An ideal I of R is called a 2-absorbing ideal if for all elements $a, b, c \in R$ such that $abc \in I$, $ab \in I$ or $bc \in I$ or $ac \in I$. Recently, the authors in [5], introduced the notion of S-prime ideal (as a generalisation of prime ideal) as follows: an ideal I of R disjoint with a multiplicative set S is called S-prime, if there exists $s \in S$ such that for all $a, b \in R$ with $ab \in I$, we have $sa \in I$ or $sb \in I$.

The notion of S-2-absorbing submodule was introduced in [6]. Let M be an R-module and S a multiplicatively closed subset of R. A submodule P of M is said to be an S-2-absorbing if $(P :_R M) \cap S = \emptyset$, and there exists a fixed $s \in S$ such that $abm \in P$ for some $a, b \in R$ and $m \in M$ implies that $sab \in (P :_R M)$ or $sam \in P$ or $sbm \in P$. In [3], the authors introduce the notion of S-n-absorbing ideals which are also generalizations of prime ideals. For a natural number n and an ideal I of R disjoint from S. Then I is an S-n-absorbing ideal if there exists $s \in S$ such that $r_1, ..., r_{n+1} \in R$ with $r_1, ..., r_{n+1} \in I$ implies $s\prod_{1 \leqslant i \leqslant n+1 i \neq j} r_i \in I$ for some $1 \leqslant i \leqslant n+1$.

The main purpose of this paper is to introduce and study the notions of S-2-absorbing ring. Let R be a ring, S a multiplicative subset of R. We say that R is S-2-absorbing if every proper ideal of R disjoint with S is an S-2-absorbing ideal.

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Approximate solution for a certain type of functional equations

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Abstract:

In 1940, S.M. Ulam proposed the famous Ulam stability problem. In 1941, D.H. Hyers solved the well-known Ulam stability problem for additive mappings subject to the Hyers condition on approximately additive mappings. The aim of this work is to give the general solution of a type of functional equations, and under some natural conditions, we discuss the stability results of such functional equation. We conclude this work by presenting some important consequences.

Key Words: *Stability; Functional equations; Approximation, Topological vector space.*

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Stability of functional inequalities in quasi-Banach spaces

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Abstract:

The stability problem of functional equations and inequalities was motivated by a question posed by Ulam in 1940 and an answer to it provided by Hyers in the following year. In this paper, we will study the generalized Ulam stability of a certain class of functional inequalities associated with Jordan-von Neumann type additive functional equation in quasi-Banach spaces.

Key Words: *Stability. Functional inequalities. Quasi-Banach spaces*

Introduction: *One of the most interesting questions in almost all areas of mathematical analysis is as follows:* When is true that a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly? *The stability problem of functional equations and inequalities originates from such a fundamental question. The first problem concerning the stability of group homomorphism was proposed by Ulam (see [5]). Hyers [3] gave a first affirmative partial answer to the problem of Ulam by proving the stability of additive mappings in Banach spaces. Since then, the stability problems of several functional equations and inequalities have been extensively investigated by a number of authors and there are many interesting results concerning this problem.*

The quasi-Banach space is one interesting generalization of Banach space. The first difference between a norm and a quasi-norm is that the modulus of concavity of a norm equal to 1*, while that of a quasi-norm is grater than or equal to* 1*. This causes the quasi-norm to be not continuous in general (see [4, Example 3]), while a norm is always continuous. However, there is a class of continuous quasi-norms, called p-norms. Moreover, for each quasi-norm there exists a p-norm that is equivalent to given quasi-norm [4]. For this reason, many authors assumed in their works that every quasi-Banach space is a p-Banach space. Recently, results for the stability of functional equations in quasi-Banach spaces which are not assumed to be p-Banach spaces have been studied (see, e.g., [1, 2]).*

Let a_i *and* b_i*,* $i = 1, 2, \ldots, l$ *(*l *is a positive integer with* $l \geq 3$*) be nonzero real numbers. The main aim of this paper is to study the generalized Ulam stability of the functional inequality*

$$\left\| \sum_{i=1}^{l} a_i f(x_i) \right\| \leq \left\| f\left(\sum_{i=1}^{l} b_i x_i \right) \right\| \tag{1}$$

in quasi-Banach spaces (which are not necessary p-Banach spaces) by using the direct method.

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The First Cohomology Group of the Units of a Real Biquadratic Field and Its Pólya Group

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Abstract:

Let $\mathcal{O}_K$ and $\mathcal{C}l(K)$ denote the ring of integers and the class group of a number field K, respectively. The Pólya group $Po(K)$ of K is the subgroup of the class group of K generated by the classes of the products of maximal ideals with the same norm. A Pólya field is a number field whose Pólya group is trivial. If K is a finite Galois extension of $\mathbb{Q}$, the Pólya group $Po(K)$ is the subgroup of the class group of K formed by the strong ambiguous classes of K. In this case, Zantema established in [5] the existence of an exact sequence linking the Pólya group, the first cohomology group of the units of K, and the ramification index of the ramified primes in K.In his paper [2], Chabert provided a general formula for the order of the Pólya group $Po(K)$ when K is a bicyclic biquadratic number field. The formula depends on the unit index of K, the number of ramified primes in K, and the number of fundamental units of norm 1 in the quadratic subfields of K. Focusing on the real case, we will discuss the cardinality of the first cohomology group $H^1(G, E_K)$ of the units of K.

Key Words: Pólya group, Pólya field, Biquadratic number field, Ambiguous ideals

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Confluent hypergeometric functions in continued fractions

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This article seeks to explore the continued fraction expansion of the confluent hypergeometric function, presenting it in detail to enhance understanding of this mathematical concept. We also aim to introduce a systematic and efficient approach for calculating the Generalized Hypergeometric series, which is often encountered in various applications across mathematics and physics. To further illustrate the theoretical principles discussed, we provide a series of numerical examples that highlight the practical implications and accuracy of our method. Through these examples, we aim to bridge the gap between theory and application, demonstrating how these mathematical tools can be utilized effectively.

Key Words: *Positive definite matrix, continued fraction, Generalized Hypergeometric series, Kummer functions, Matrix continued fractions, Convergence criteria.*

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On the structure of quotient quasi-rings

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Abstract:

We consider N to be a 3-prime field and P to be a prime ideal of N. In this paper, we study the commutativity of the quotient near-ring N/P with left multipliers and derivations satisfying certain identities on P, generalizing some well-known results in the literature. Furthermore, an example is given to illustrate the necessity of our hypotheses.

Key Words: *3-prime near-rings, Prime ideals, Additive Maps*

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Artificial intelligence

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An Innovative Approach Utilizing Artificial Neural Networks and the Hybrid GA-IPA Technique to Solve Fractional Non-linear Dynamical System

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Abstract:

This study explores the dynamics of a non-linear prey-predator system using artificial neural networks, a mathematical modeling tool widely applied in various scientific and engineering fields. The aim of this paper is to introduce a novel approach for solving fractional ordinary non-linear prey-predator systems. To achieve this, we employ a neural network framework combined with the hybrid GA-IPA method, which integrates a genetic algorithm with an interior point algorithm. Additionally, we present numerical results to validate the proposed ANN-GA-IPA approach for tackling fractional non-linear prey-predator systems.

Key Words: *Dynamical systems, Non-linear prey-predator system, Fractional calculus, Artificial nueral network, Interior point algorithm, Genetic algorithm, Adams Bashfourth method*

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Improved Clustering with Fractional Fuzzy C-Means: via Long-Memory Effects and Geometric Indices.

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Abstract:

The fuzzy C-means (FCM) clustering algorithm is a widely used unsupervised learning method known for its ability to identify natural groupings within datasets. While effective in many cases, FCM faces challenges such as sensitivity to initial cluster assignments, slow convergence, and difficulty in handling non-linear and overlapping clusters. Aimed at these limitations, this presentation introduces a novel fractional fuzzy C-means (Frac-FCM) algorithm, which incorporates fractional derivatives into the FCM framework. By capturing non-local dependencies and long memory effects, fractional derivatives offer a more flexible and precise representation of data relationships, making the method more suitable for complex datasets. Additionally, a genetic algorithm (GA) is employed to optimize a new least-squares objective function that emphasizes the geometric properties of clusters, particularly focusing on the Fukuyama-Sugeno and Xie-Beni indices, thereby enhancing the balance between cluster compactness and separation.

Key Words: *Fuzzy c-means, fractional derivative, genetic algorithm.*

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An Innovative Dropout Strategy for Enhancing Robustness in GNNs

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Abstract:

Recently, Graph Neural Networks (GNNs) have emerged as a powerful tool for graph representation and learning tasks. The core of GNNs is the message propagation mechanism across a node's neighbors. However, standard GNNs still suffer from limitations such as lack of robustness, over-smoothing, and overfitting. In this study, we propose an innovative dropout technique that involves selecting a certain percentage of edges to participate in message passing after a message propagation layer. This approach aims to avoid data deterioration and mitigate the issue of non-robustness. Experiments were conducted to confirm the efficiency of the proposed scheme.

Key Words: *Graph Neural Network, robustness, over-smoothing, node classification.*

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Bridging AI and Culture: Developing an Autonomous Robot with Darija Conversational Capabilities and Object Recognition

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Abstract:

This paper presents the design and implementation of a versatile autonomous robot that combines advanced artificial intelligence capabilities for navigation, object recognition, and conversational interaction in Moroccan Darija. Designed for modularity, the system aims to be adaptable to various domains such as healthcare, education, and personal assistance. The robot is built on a Raspberry Pi 5, offering a compact, cost-efficient, and scalable hardware platform capable of supporting real-time AI processing, making it accessible for a wide range of applications.
For conversational interaction, the project leverages Atlas-Chat, a cutting-edge large language model fine-tuned specifically for Darija. To further enhance its performance, we will fine-tune the model on task-specific datasets, enabling it to respond accurately and contextually in real-world scenarios. This customization ensures effective domain-specific interactions, making the robot a culturally and linguistically inclusive solution.
The vision system will integrate advanced object detection capabilities. To ensure optimal performance, we will benchmark state-of-the-art models, including YOLOv8, DETR, and other CNN-based approaches, evaluating their accuracy and efficiency for deployment on the Raspberry Pi 5. The chosen model will enable the robot to recognize objects in its surroundings and incorporate this understanding into conversations. For instance, when asked, "What do you see in front of you?" the robot will provide a contextually relevant response such as, "I see an apple."
This project aims to bridge the gap between AI-powered robotics and cultural inclusivity while addressing practical constraints such as affordability and scalability. By focusing on modularity and customization, the system offers potential applications across diverse industries, from assisting medical professionals to enhancing educational experiences, making it a flexible solution for a variety of real-world challenges.

Key Words: *Autonomous robot, AI, Moroccan Darija, object recognition, Raspberry Pi, conversational interaction, cultural inclusivity, modularity*

Introduction: *This paper discusses the design and implementation of an autonomous robot that uses artificial intelligence to navigate, recognize objects, and interact conversationally in Moroccan Darija. The robot's modular design aims to cater to various domains such as healthcare, education, and personal assistance, providing a scalable, cost-efficient solution powered by a Raspberry Pi 5. The integration of advanced AI models for language processing and object detection ensures that the robot can interact intelligently and contextually, making it suitable for real-world applications.*

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Investigating the Memory Effect in Supply Chain Management Using Fractional Calculus Models

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Abstract:

Fractional calculus is one of the oldest concepts of applied mathematics. This concept is regarded at first as an abstract because the mathematical theory was there but with no clear applications; his role was only reduced to extending the traditional concept of differentiation and integration beyond integer orders. It took time for researchers to find meaningful interpretations of fractional derivatives and integrals before discovering the unique ability of fractional calculus to model complex behaviors that traditional calculus cannot easily capture, especially in systems where memory, hereditary properties, or anomalous dynamics are involved. Since then, fractional calculus has a wide spread in various fields. In this conference, we will discuss the use of fractional calculus in the domain of supply chain management, focusing on the paper "Enhancing Symmetry and Memory in the Fractional Economic Growing Quantity (FEGQ) Model " as a case study. We will demonstrate the effectiveness of using fractional calculus, particularly in incorporating the concept of memory effects. We will illustrate different methods used to solve those differential equations. Numerical application results are highlighted, in which we indicate the importance of the memory effect in choosing the optimal business policy to achieve the optimal profit. We will conclude by exploring the limitations of the use of fractional calculus in both our work and related research.

Key Words: *Fractional differentiation and Fractional Integration; Memory dependent derivative.*

Introduction: .....

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A novel fuzzy clustering approach for robust MRI brain segmentation

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Abstract:

Image segmentation is one of the most important and challenging processes in medical image analysis, particularly for Magnetic Resonance Imaging (MRI) brain scans. Accurate segmentation of brain tissues, such as gray matter, white matter and cerebrospinal fluid, is essential for diagnosing neurological disorders and conducting research. However, traditional segmentation methods often struggle with noise, intensity inhomogeneity, and the overlapping nature pf tissue intensities in MRI data. In this context, we propose a novel approach to MRI brain segmentation using fuzzy clustering, which addresses these challenges by incorporating spatial information and handling uncertainty in voxel classification. Experiments were conducted on publicly available MRI datasets, and results show that the proposed fuzzy clustering approach outperforms traditional FCM and other state-of-the art methods in terms of segmentation accuracy and robustness to noise. The proposed algorithm demonstrates significant potential for clinical applications where accurate brain tissue segmentation is crucial.

Key Words: processing, Magnetic resonance images, Brain tissue segmentation, Noise, Fuzzy clustering. Convergence analysis.

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An Innovative Intrusion Detection Model for VANETs Using Convolutional Neural Networks

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Abstract:

intelligent transportation systems (ITS) are booming, offering innovative services to improve the safety and efficiency of transportation networks. An essential component of these systems is the vehicular ad-hoc network (VANET), aimed at reducing collisions and securing information in real time. Security challenges in VANETs are paramount, and intrusion detection systems are crucial to identifying malicious activity. The integration of artificial intelligence (AI) into these systems has shown significant improvement in attack detection, in this paper We propose a new network intrusion detection model using Convolutional Neural Networks (CNN) for multi-class classification and the Experimental results show that the proposed model is capable of automatically identifying and classifying multiple types of intrusions

Key Words: Intelligence, vehicular ad hoc networks (VANETs), Intrusion detection system, Security.

Introduction: learning techniques have been widely used for intrusion detection, but they have several limitations, such as a high rate of false positives and an inability to adapt to new threats. To overcome these challenges, researchers are turning to more advanced approaches, such as deep learning. In particular, convolutional neural networks (CNNs) are promising in this field. Initially used for image recognition and natural language processing, CNNs have proven effective in cybersecurity due to their ability to automatically extract complex features and analyze network traffic to detect anomalies, In this paper, we propose An Innovative Intrusion Detection Model for VANETs based CNN that can effectively detect a range of attacks

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Fuzzy-PID controller

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Abstract:

The rapid increase in vehicular traffic has introduced major challenges in road safety, congestion, and environmental pollution. Vehicle platooning, a strategy where multiple vehicles travel in close proximity under coordinated control, has emerged as a promising solution to improve traffic flow and safety. This paper discusses an innovative adaptive fuzzy control technique specifically designed for high-speed vehicular platoons, addressing the critical challenge of communication delays that can negatively impact platoon stability and performance. The proposed control architecture combines fuzzy logic with traditional Proportional-Integral-Derivative (PID) control, harnessing the advantages of each approach. Fuzzy logic enables effective handling of the non-linear dynamics and uncertainties typical in vehicular systems, while PID control ensures accurate maintenance of desired inter-vehicle time gaps. This hybrid approach allows for real-time adaptation to changing traffic conditions, vehicle dynamics, and communication delays, which are common in real-world scenarios.

Key Words: *smart vehicle; platooning; power modeling, fuzzy, connected automated vehicles*

Introduction: .....

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Control Theory

5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

On The Controllability of a class of Time-Fractional Systems with order between 1 and 2.

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Abstract:

The primary goal of this paper is to investigate the controllability of linear time-fractional systems involving the Caputo fractional derivative of order $1 < \gamma < 2$. The paper focuses on developing a theoretical framework to demontrate some exact and approximate controllability properties of Time-fractional systems by using the new expression of controllability operator. Next, we expand by giving the steps of the attempted approach (HUM) for finding the minimum energy control, which steers the system from the initial to a target function. Furthermore, to demonstrate the efficacy of our findings, we provide a few numerical simulations of one-dimensional systems using a zonal actuator or pointwise actuator.

Key Words: *Time-fractional systems; Controllability; Caputo derivative; HUM Approach; Control theory; Cosine and sine family.*

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Strong stabilization for a class of bilinear systems of neutral type with discrete multi-delays

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Abstract:

This work addresses the problem of polynomial stabilization for an abstract class of homogeneous bilinear systems of neutral type with discrete multi-delays, evolving on a real Hilbert space. Sufficient conditions in term of observation estimates are formulated to achieve the strong stabilization with polynomial decay estimate via an explicit sequence of feedback controls. Additionally, through an appropriate decomposition of the state space, we introduce a new sequence of feedback controls that depends only on the state projection onto a suitable subspace, achieving the polynomial stabilization of the system at hand. Finally, we present examples and numerical simulations to illustrate the effectiveness of the obtained results.

Key Words: *Neutral systems, Multi-delays, Feedback control, Strong stabilization, Decay estimate, Decomposition method.*

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Observability for a Class of Ψ-time -fractional linear systems.

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Abstract

This study aims to introduce the concept of observability for a specific class of linear time-fractional systems within the Ψ-Caputo framework, characterized by a differentiation order between 1 and 2, and without requiring prior knowledge of the initial state vector. The primary objective is to define observability and explore its fundamental properties. To achieve this, we reconstruct the system's initial state vector using an extended version of Hilbert's Uniqueness Method (HUM), which simplifies the problem into a more manageable form and facilitates the development of an algorithm for state computation. Furthermore, practical examples are provided by varying the Ψ function to validate the theoretical findings. The effectiveness of the proposed algorithm is demonstrated through comprehensive numerical simulations presented in the concluding section.

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Stabilization and decay estimate for a class of neutral functional differential equations.

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Abstract
We deal with the problem of stabilization for a class of retarded distributed bilinear systems of neutral type evolving in a Hilbert space. We first give a proof for the existence and uniqueness of the solution for the considered systems.Then, under a null controllability condition, we establish the stabilization result and provide an explicit estimate of the energy decay and weak stabilizability. Finally, we present some illustrating applications to parabolic and hyperbolic.

Introduction
We consider the question of strong stabilization for a class of distributed bilinear systems of neutral type with time delay $r > 0$ described as follows :

$$\begin{cases} \dfrac{d\mathcal{D}z_t}{dt} = A\mathcal{D}z_t + v_r(t)Bz(t-r), & t \geq 0, \\ z(t) = \varphi(t), & t \in [-r, 0]. \end{cases} \tag{1}$$

where

- **i)** $A : \mathcal{D}(A) \subset H \to H$ is the infinitesimal generator a strongly continuous semigroup of contractions $\{S(t)\}_{t\geq 0}$
- **ii)** The control operator $B : H \to H$ is a bounded linear operator and $t \longmapsto v_r(t)$ is a scalar function which represents the control.
- **iii)** The operator $\mathcal{D} : \mathscr{C} \to H$ is defined by $\mathcal{D}(\phi) = \phi(0) - L\phi(-r), \forall \phi \in \mathscr{C}$, where $L : H \to H$ is a linear bounded operator such that $\|L\| < 1$.

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Optimal feedback control stabilisation for a class of bilinear delayed systems of neutral type

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Abstract:

This paper focuses on the problem of strong stabilisation for an abstract class of distributed bilinear delayed systems of neutral type, evolving in real Hilbert state space. A bounded feedback control is constructed and sufficient conditions are investigated to ensure strong stabilisation with a polynomial decay estimate of the stabilised state. The stabilising control is characterized through a minimization problem. Moreover, under an appropriate decomposition of the state space, we consider a feedback control that depends only on the state projection on a suitable subspace, and guarantee the polynomial stabilisation for parabolic like systems. Finally, examples with numerical simulations are presented to illustrate the effectiveness of the obtained results.

Key Words: *Bilinear systems; Neutral systems; Optimal control; Strong stabilisation; Decay estimate; Decomposition method.*

Introduction: *This paper is concerned with the feedback stabilisation question of the following infinite dimensional bilinear neutral system*

$$\begin{cases} \dfrac{d}{dt}\mathcal{D}y_t = A\mathcal{D}y_t + v(t)By(t), & t \geq 0, \\ y(t) = \varphi(t), & t \in [-r, 0], \end{cases} \tag{1}$$

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ROBUST REGULATION OF AN AEROBIC WASTEWATER TREATMENT PROCESS MODEL

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Abstract:

In this work, we intend to develop a robust regulation of an aerobic wastewater process model. The model contains two on line measured state variables and, due to exogenous disturbances and functional uncertainties of the kinetics, two unmeasured state variables. The first element is to find an interval observer of the unmeasured state variables. The second element consists to build a feedback control, using the dilution and aeration rates as controls, to stabilize the output variables around a suitable value.

Key Words: *Interval observers; feedback control; robust partial stabilization.*

Introduction: *The model is based on four fundamental mass balance equations aiming at developing the processes by separating the effects.*

$$\begin{cases} \dfrac{dS(t)}{dt} = -\dfrac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \\ \dfrac{dDo(t)}{dt} = -K_0\dfrac{\mu(t)}{Y}X(t) - D(t)(1+r)Do(t) + D(t)Do_{in} + \alpha W[Do_{max} - Do(t)] \\ \dfrac{dX(t)}{dt} = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \\ \dfrac{dX_r(t)}{dt} = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \end{cases} \tag{1}$$

In the above equations the following notations where used :
$X(t)$ *- biomass ,*
$S(t)$ *- substrate,*
$Do(t)$ *-dissolved oxygen,*
Do_{max} *-maximum amount of dissolved oxygen,*
$X_r(t)$*-recycled biomass,*
$D(t)$*-dilution rate (the ratio between the flow rate of the influent and the volume of the aeration tank),*
S_{in} *and* Do_{in} *- concentrations of dissolved oxygen and of substrate in the mass of the influent,*
Y *biomass yield factor,*
α *- oxygen transfer rate,*
W *-aeration rate,*
K_0 *- model constant,*
r *- the ratio between the re-circulated flow rate and the influent flow rate*
β *- the ratio between the waste flow rate and the influent flow rate*
we supposed that The biomass growth rate μ *depends on the concentrations of substrate, on the concentrations of dissolved oxygen and on a few kinetic parameters, according to the model suggested by Olsson :*

$$\mu = \mu_{max}\frac{S(t)}{K_s + S(t)}\frac{Do(t)}{K_{Do} + Do(t)}$$

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Indirect controllability of coupled degenerate Euler-Bernoulli beam equations

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Abstract:

The aim of this talk is to present a new result on the null controllability property for two coupled degenerate beam equations where the control function acts only on one of the two equations. The beam equations are coupled via velocity damping terms, where the velocity of the second equation appears in the first equation and viece versa. As a first step, we analyze the well posedness of the associated homogeneous adjoint problem in suitable functional spaces using semigroup theory. Then, we derive the so-called observability estimate for this kind of problem in a sufficiently large time, provided that the coupling coefficient is assumed to be positive and small. By means of this estimate and applying the classical Hilbert uniqueness method, we deduce that the original system is null controllable. We also give the explicit controllability time which depends both on the degree of the degeneration and the coupling term.

Key Words: *Controllability, coupled system, degenerate fourth order equations, HUM*

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Neural Network Controllability of financial Dynamic Systems

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Abstract:

Using neural network-based methods, this work investigates the controllability of nonlinear dynamic systems, particularly those influenced by sigmoid activation functions and related to financial problems. The objective is to demonstrate that control techniques can minimize discrepancies between the system's actual output and a target output or equilibrium point. The approach follows several key steps: first, the system is linearized to facilitate an analysis of state stability and controllability. Next, Pontryagin's maximum principle is applied to theoretically assess controllability and identify the optimal control function. Simulations are performed to validate these findings, employing control approximations through recurrent neural networks (RNNs) and Long Short-Term Memory networks (LSTMs). Finally, a practical application in financial time series forecasting is presented, using the AAPL dataset to illustrate the method's effectiveness in reducing prediction error. This study offers a solid theoretical framework for controlling nonlinear systems and opens promising avenues for complex system management in fields such as finance.

Key Words: *Dynamic systems, controllability, stability, linearization, lyapunov function neural network, recurrent neural networks, Hopfiled networks, optimal control.*

Introduction: *In several disciplines, including biology, engineering, and robotics, dynamic system control is crucial. The nonlinear differential equations governing these systems require a robust and often intricate theoretical framework to ensure the existence of solutions, their stability, and the performance optimization or control needed to achieve specific objectives. The complexity of these systems dynamics poses several challenges in research, particularly when they are subjected to unknowns, disturbances, or hard-to-model parameters especially when investigating real-world phenomena that evolve over time. In recent decades, new techniques based on artificial neural networks have emerged, providing powerful tools for modeling and managing these complex systems. Neural networks have become the method of choice for analyzing the behavior of dynamic systems due to their ability to learn nonlinear relationships and generalize from data. They not only capture intricate connections between a system's states but also adapt to dynamic and unpredictable environments.*

Inspired by the functioning of the human brain, neural networks were developed in the 1950s. These networks form an information-processing system composed of several interconnected neurons. Among the first neural networks, the perceptron model, developed in 1958 by Frank Rosenblatt, was designed for binary classification tasks. However, interest in this technology declined during the 1970s due to the perceptron's limited ability to solve nonlinear problems, despite its initial promise. A resurgence of interest in neural networks did not occur until the late 1980s and early 1990s, thanks to the backpropagation algorithm developed by David Rumelhart and James McClulloch. This algorithm enabled efficient adjustments of connection weights between neurons, allowing for the learning of deeper and more complex networks. Subsequently, the application of neural networks expanded to various fields, including computer vision, speech recognition, and natural language processing, facilitated by new architectures such as recurrent neural networks (RNNs) for sequence processing and convolutional neural networks (CNNs) for image processing. With the rise of big data and advancements in computational power, deep neural networks, also known as deep learning, have become prevalent in artificial intelligence applications. These technical advancements have empowered neural networks to learn intricate representations and generalize effectively from large datasets. When defined and trained correctly, these networks can perform tasks such as pattern recognition, classification, and, most notably, dynamic system control, opening up new possibilities for managing and forecasting complex behaviors.

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with norm $\|x\|_1 = \sum_{i=1}^{n} |x_i|$ and let the superscript T denote the

transposition.
In this work, the following state equation can be used to characterize the dynamic behavior of neural networks :

$$\dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij}(t) f_j(x_j(t)) + I_i \qquad i = 1, 2, ..., n \tag{1}$$

where n corresponds to the number of units in the neural network; $x_i(t)$ corresponds to the state vector to the ith unit at time t; $f(x(t)) = f_j(x_j(t)) = (f_1(x_1(t)), f_2(x_2(t)), ..., f_n(x_n(t)))$ represents the activation function of the neurons at time t; a_i represents the self-inhibition with wich the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; the external bias on the ith neuron is represented by I_i; $b_{ij}(t)$ indicates the strength of the link between the jth and ith neurons, multiplying a firing rate in unit j by b_{ij} yields the repeating rate of increase or decrease in the average membrane potential in unit i, positive values of b_{ij} reflect aggregates j whose net average effect on aggregate i is excitatory and negative b_{ij} correspond to inhibition.
Our objective is to linearize the previous system, compute equilibrium point to prove its stability and compute the optimal control.

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Strong stabilization and decay estimate for a class of semilinear time-delay systems of neutral type

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Abstract:

In this paper, we deal with the problem of feedback stabilization for a class of nonhomogeneous semilinear time-delay systems of neutral type evolving on a Hilbert space. To achieve this goal, and based on the decomposition of the state space, we propose a bounded feedback control that depends only on the state projection on an appropriate subspace to study the strong stabilization. Sufficient conditions are formulated in terms of observability like assumptions. In addition, the rate of strong stabilization is explicitly given. Finally, illustrative examples are presented.

Key Words: *Semilinear systems, Non homogeneous systems, Time delay, Neutral systems, Strong stabilization, Polynomial stabilization, Decay estimate.*

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Contraction-Based Hidden State Regularization in Convolutional Neural Networks

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Abstract:

Convolutional neural networks (CNNs) are widely utilized across various research domains for their powerful capability to model complex structures. Unlike other artificial neural networks, CNNs incorporate convolutional operations that enhance network performance directly. This distinctive feature has led to the development of numerous convolutional models and techniques. CNNs are characterized by a sophisticated architecture with convolutional layers for feature extraction and fully connected layers for higher-level reasoning and classification. This structure allows CNNs to effectively capture hierarchical patterns and use learned features for accurate decision-making. Nonetheless, CNNs can be sensitive to slight input variations. This study presents an innovative regularization approach based on Contraction Theory Analysis to improve CNN stability. Specifically, this regularization approach ensures that the fully connected layer outputs for inputs of the same class converge closely, enhancing robustness in classification.

Key Words: *Convolutional Neural Networks (CNNs), Contraction Theory, Regularization, Feature Extraction, Stability.*

Introduction: *In recent years, convolutional neural networks (CNNs) have gained significant attention due to their outstanding performance across various fields, such as image and signal processing [14], natural language processing [10], and speech recognition [1]. CNNs are particularly notable for their ability to represent complex structures, largely enabled by convolutional operations [2]. Typically, these deep architectures utilize convolution operations to extract features from extensive input data sets [5]. In each convolutional layer, filtersâalso known as kernelsâscan the input images to generate multiple feature maps [7]. Despite their strengths, CNNs face a critical challenge: they are highly sensitive to input data variations, which can lead to misclassification from slight perturbations [6]. Regularization techniques are, therefore, essential to mitigate such perturbations and prevent overfitting. For instance, to minimize data disturbance in output layers, Jacobian regularization has been implemented by the authors in [11, 13]. Similarly, Lipschitz regularization was applied in [4] to control the Lipschitz constant, thereby managing the effects of input perturbations on neural network gradients. Among well-known regularization techniques, DropConnect [9] resembles DropOut [3] and seeks to reduce overfitting by modifying network behavior; DropOut deactivates certain neurons, while DropConnect randomly zeros specific weight values. Regularization can also be applied within hidden layers, as seen in Manifold-mixup [8], where hidden layer outputs are combined to improve neural network robustness. Contraction theory has also proven effective in addressing data perturbations [13]. Building on these methods, the authors in [12] enhanced deep neural networks by developing more contractive activation functions. This paper proposes a novel regularization technique based on contraction theory, applied to each layer in the FullyConnected section. Specifically, we introduce weight adjustments in the hidden layers using target centers, enabling layer outputs for data inputs in FullyConnected to converge closely when they belong to the same class.*

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pseudo almost periodic solution of a Delayed Quaternion-Valued Fuzzy Recurrent Neural Networks model on Time Scales

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Abstract:

we consider quaternion-valued fuzzy recurrent neural networks with time-varying delays on time scales. Different from the previous literature, we use a direct method to obtain our theoretical results to avoid decomposing the model into real-valued or complex valued systems. Then, we obtain some sufficient conditions on the existence, uniqueness, and Sp-global exponential stability of weighted Stepanov-like pseudo almost periodic solution on time scales of the considered model by applying inequality analysis techniques on time scales, a fixed point theorem, and composition theorem, and by constructing an appropriate Lyapunov function. At the end of this work, we give a numerical example and simulations to illustrate the effectiveness of the obtained results.

Key Words: Time scales, Quaternion-valued recurrent neural networks, Fuzzy theory,Weighted Stepanov-like pseudo almost periodicity, global stability.

Introduction: As we know, discrete and continuous recurrent neural networks play a key role in theoretical research and applications. Also, discrete-time neural networks are more beneficial and convenient for numerical simulation and computation than continuous-time NNs. Hence, not only do we need to study continuous-time neural networks, but we also need to study discrete NNs. To avoid the difficulties of studying the dynamic properties of continuous and discrete systems, respectively, it is helpful to study these properties on time scales, which Stefan Hilger ([1]) introduced in his PhD thesis in order to unify continuous and discrete analysis. As a result, using time scale dynamic systems, subjects such as the existence of a solution, stability, floquet theory, periodicity, and the dynamics of NNs can be studied more precisely and broadly. Recently, the existence and stability of the periodic solution on time scales has been one of the most attractive themes in the context of various kinds of abstract dynamic equations , partial dynamic equations ([2]), integro-dynamic equations ([3]) and general dynamic systems ([4]).
Motivated by the above statement, we summarized the innovation points of this paper as follows : (I) we integrate fuzzy operations into quaternion-valued RNNs with time-varying delays on time scales. (II) For the time being, this is the first time that theWSpPAP dynamics of a delayed QVFRNNs are being investigated on time scales, which can unify both continuous time and discrete time cases of RNNs. The QVFRNNs proposed in this work also contain real-VFRNNs and complex- VFCNNs as their special cases. (III) We take into account another oscillation space that has never been taken into account in the different classes of recurrent neural networks.

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

TITLE

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Abstract:

This paper is concerned with the dynamical behaviors of almost periodic solution of Lotka-Volterra neural networks. Based on the Banach's fixed point theorem, exponential dichotomy of linear differential equations, and almost periodic functions theory, a set of sufficient criteria that guarantee the existence and uniqueness of a positive almost periodic solution are discussed. Moreover, numerical simulations are carried out to illustrate the validity of the theoretical results.

Key Words: *Lotka Vlterra neural networks; Almost periodic solution*

Introduction: *Over the last few decades, there have been significant advances in the field of neural network research. Various neural networks, such as BAM neural cellular networks, recurrent neural networks, Memristive neural networks, and Lotka-Volterra neural networks, have been extensively researched and successfully applied in a wide range of applications. The well-known Lotka-Volterra neural network (LVNNs) has recently attracted special attention in research because it is sufficiently general to include several well-known neural networks as special cases, and has promising application potential for classification tasks, associative memory problems, parallel computing, and nonlinear optimization.*
On the other hand, periodic movement is common in a wide range of real-world phenomena, including in the biological system, the periodic oscillation of the human brain, climate change in the four seasons, vibrations, and so on. Therefore, a growing number of researchers have recently investigated the periodicity of Lotka-Volterra neural networks. Because the dynamic behaviors of Lotka-Volterra neural networks are important in their applications and implementation, several researchers have investigated the periodicity, stability, synchronization, dissipativity, and other properties of Lotka-Volterra neural networks.

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Directional convexificators in fractional multiobjective optimization

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Abstract:

Necessary and sufficient optimality conditions are investigated for fractional multiobjective problem. These optimality conditions are given in terms of directional upper (upper semi-regular) convexificators. We reach our necessary optimality results by using a weaker type Abadie constraints qualifications. Sufficient optimality conditions are given under a generalized convexity. The obtained results are illustrated by some examples.

Key Words: *Multiobjective fractional optimization; Directional upper convexificators; Directional upper semi-regular convexificators; Constraint qualification; Optimality conditions.*

Introduction: *Our aim in this work is to derive necessary and sufficient optimality conditions for nonsmooth fractional optimization problem (P) by using the concept of directional convexificators, which can be more advantegous than using convexificators, since directional convexificators can be strictly included in convexificators (see [4]). To our knowledge, this paper is the first to use this notion of directional convexificators for fractional optimization problem.*
The outline of this paper is as follows. Section 2, contains basic definitions and preleminary material from nonsmooth analysis. Section 3 is devoted to find necessary optimality conditions under a directional Abadie-type constarint qualification. In section 4 we first present a generalized convexity in terms of directional upper (upper semi regular) convexificators, then we propose sufficient optimality conditions. Some examples are given to illustrate our findings.

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FEEDBACK STRONG STABILIZATION FOR A SEMILINEAR CONTROL SYSTEM OF TIME-DEPENDENT DELAY

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Abstract:

This paper explores the feedback strong stabilization of semilinear systems with a time-dependent delay that exhibits weak observation properties. Some illustrated examples are given.

Key Words: *feedback Strong stabilization, delayed semilinear systems, feedback control*

Introduction: *In this paper, we are concerned with the feedback strong stabilization of the homogeneous delayed semilinear system. where the state space is a Banach space X , the dynamic A is an unbounded operator with domain $D(A) \subset X$ and generates a C_0-semigroup on X. The control operator B is a linear bounded operator from X into X.*

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THE CONTROLLABILITY OF FRACTIONAL DIFFERENTIAL SYSTEM WITH STATE AND CONTROL DELAYS

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Abstract:

In this research, we examine the controllability of linear fractional differential control systems with delays in both state and control by using RiemannâLiouville fractional derivatives. Utilizing an explicit solution formula, we establish a rank criterion for controllability. We provide necessary and sufficient conditions for the controllability of these fractional differential systems. Finally, a numerical example is presented to illustrate and support the findings.

Key Words: *Controllability; Delayed Mittag-Leffler type matrix; State delay; Control delay*

Introduction: *fractional differential equation is a mathematical model which is useful for the explanation of hereditary characteristics and memory of different processes and materials. A variety of research work is based on the basic study of fractional differential equations as in further work various researchers considered control problems. The controllability shows a major presence in the advancement ofmodernmathematical control theory and engineering which has a close connection with structural decomposition, quadratic optimal and so on. Controllability is a qualitative property of fractional delay dynamical system, so one needs to find its representation of a solution. He and Wei gave a representation of a solution and discussed the controllability and then for a fractional control delay system obtained necessary and sufficient conditions, Nirmala give a representation of a solution by using Laplace transform and Mittag-Leffler function and established controllability criteria for fractional delay dynamical system. Moreover, Khusainov et al. obtained the representation of a solution of a Cauchy problem for a linear differential equation with pure delay by using the delayed Mittag-Leffler function, Shukla et al. discussed the complete and approximate controllability of semilinear stochastic systems with delays in the state and control function with non-Lipschitz coefficients, the Schauder fixed point theorem, sequence methods and by the theory of the strongly continuous z-order cosine family, and the fixed point theorem, respectively.*

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Strong Stabilization of an Unbounded Bilinear System with Time Delay

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Abstract:

In this work, we investigate the problem of homogeneous distributed stabilization for unbounded bilinear systems with time delay. The dynamic is a linear unbounded operator, generating a contraction C_0-semigroup on a Hilbert space H. The control operator is unbounded in some sense. We derive sufficient conditions for strong stabilization through a family of bounded feedback control. Finally, these results are applied to a Heat equation with delay.

Key Words: *Homogeneous delayed bilinear systems, unbounded operator, delay feedback control, strong stabilization*

Introduction: *Bilinear systems are natural models of numerous real-world dynamical processes. They arise in many fields, such as engineering, biology, and economics. Therefore, understanding the impact of delay on the system's stability is crucial.*
In this paper, we investigate the problem of distributed stabilization for unbounded bilinear systems with time delay, described by the equation

$$\begin{cases} \dfrac{\partial z(t)}{\partial t} &= Az(t) + +v_r(t)Bz(t-r), \quad \forall t > 0, \\ z(t) &= \psi(t),\ \forall t \in [-r, 0], \\ z(0) &= z_0 = \psi(0)\cdot \end{cases} \tag{1}$$

With $r > 0$ is a fixed delay parameter. A is the infinitesimal generator of a linear C_0-semigroup of contractions on a real Hilbert space H. The memory function $\psi \in \mathcal{C} := \mathcal{C}([-r, 0], H)$. The state function $z(t)$ represents the mild solution of (1), while the real-valued function $v_r(\cdot)$ serves as the control. The linear control operator B is unbounded in some sense.
We establish sufficient conditions for the strong stabilization of the homogeneous bilinear system with delay (1).

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Cryptography and Coding Theory

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ON GREENBERG'S CONJECTURES

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Abstract:

Motivated by the following conjecture:
Conjecture **(Greenberg)** For any odd prime p and any natural number t, there exists a p-rational field whose Galois group on $\mathbb{Q}$ is isomorphic to $(\mathbb{Z}/2\mathbb{Z})^t$
Definition: F is said to be p-rational when the Galois group $G(F_S/F)$ is a free pro-p-group, where F_S is the maximal p-extension of F which is unramified outside the primes above S.
Several recent works have led to proofs of the p-rationality of certain bi-quadratic and tri-quadratic fields, and our aim is to extend and generalize these results.

Key Words: *multi-quadratic fields,p-rationnel*

Introduction: *Multi-quadratic p-rational fields give examples of number field extensions. Their structure is determined by number field theorems, with applications in number theory and cryptography. Their algebraic and arithmetic properties are studied to understand their behavior in various mathematical contexts.*

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On a problem of Pillai involving S-units and Perrin numbers

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Abstract:

The equation known as Pillai's equation, given by

$$a^x - b^y = c, \tag{1}$$

where $a > 1$, $b > 1$, and c are fixed integers, has garnered attention due to its potential to generate several non-negative integer solutions x, y. This interest stems from Pillai's research. The problem remains open; however, the special case $c = 1$, known as Catalan's conjecture, was resolved by Mihailescu. In this work, we discuss certain aspects related to Pillai's problem. Let $\{P_n\}_{n\geq 0}$ represent the Perrin sequence. We consider the exponential Diophantine equation

$$P_n - 2^x 3^y = c, \tag{2}$$

where $n, x, y \in \mathbb{Z}_{\geq 0}$. We treat the cases $c \in -\mathbb{N}$, $c = 0$, and $c \in \mathbb{N}$ separately. For the cases where $c \in \mathbb{N}$ and $c \in -\mathbb{N}$, we find all integers c such that the Diophantine equation has at least three solutions. These cases are handled independently, as we employ different techniques to prove each case.

Key Words: *Lucas numbers; S-units; linear forms in logarithms; p-adic numbers; Pillai's problem.*

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Elliptic curves over the ring $\mathbb{F}_q[e], e^3 = e$

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Abstract:

In this work, we will study in characteristic greater than or equal to 5, an elliptic curve given by the projective Weierstrass equation $Y^2Z = X^3 + aXZ^2 + bZ^3$ where the coefficients a and b are in the quotien ring of the polynomial ring with coefficients in a finite field $\mathbb{F}_q$ where q is a power of a prime number $p \geq 5$ by the polynomial $X^3 - X$. In a first time, we give some basic algebraic properties of this ring. In addition, by defining three crucial mappings denoted $\pi_i, i = 0, 1, 2$, we will be able to prove the group structure of this elliptic curve by given its isomorphic group.

Key Words: *Finite field, Finite ring, Local ring, isomorphism, Elliptic curve, Cryptography.*

Introduction: *Let $\mathbb{F}_q[X]$ be the polynomial ring with coefficients in a finite field $\mathbb{F}_q$ where q is a power of a prime number greater than or equal to 5, and let $\mathbb{F}_q[X]/\langle X^3 - X\rangle$ be its quotient ring by the polynomial $X^3 - X$. In [2], we have studied the elliptic curve $E_{a,b}(\mathbb{F}_q[e])$ defined over the nonlocal ring $\mathbb{F}_q[e] := \mathbb{F}_q[X]/\langle X^2 - X\rangle$ where $e^2 = e$ and $(a, b) \in (\mathbb{F}_q[e])^2$, and we have proving that this elliptic curve is isomorphic to the product $E_{\pi_0(a),\pi_0(b)}(\mathbb{F}_q) \times E_{\pi_1(a),\pi_1(b)}(\mathbb{F}_q)$ of the elliptic curves $E_{\pi_0(a),\pi_0(b)}(\mathbb{F}_q)$ and $E_{\pi_1(a),\pi_1(b)}(\mathbb{F}_q)$ where π_0 and π_1 are the mappings defined by*

$$\begin{array}{ccc} \pi_0 : \mathbb{F}_q[e] & \longrightarrow & \mathbb{F}_q \\ x_0 + x_1 e & \longmapsto & x_0 \end{array} \quad \text{and} \quad \begin{array}{ccc} \pi_1 : \mathbb{F}_q[e] & \longrightarrow & \mathbb{F}_q \\ x_0 + x_1 e & \longmapsto & x_0 + x_1 \end{array} .$$

In this paper our objective is to extend the previous study to the ring $\mathbb{F}_q[e]$ where $e^3 = e$. Precisely, we will study the elliptic curve given by the Weierstrass equation $Y^2Z = X^3 + aXZ^2 + bZ^3$ where a and b are in the finite ring $\mathbb{F}_q[X]/\langle X^3 - X\rangle$. As a result, we will show that this curve is isomorphic to a finite product of elliptic curves defined over the finite field $\mathbb{F}_q$ by the same Weierstrass equation with particular coefficients in $\mathbb{F}_q$, and then the addition law can be explicitely calculated using the explicit formulae of addition law for an elliptic curve defined over a field given by Bosma and Lenstra in [1].

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A new SRTOMS algorithm for 5G New-Radio Standards

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Abstract:

Error-correcting codes, especially Low-Density Parity-Check (LDPC) codes, are crucial for maintaining the reliability and integrity of data transmission in modern communication systems. In the context of 5G New-Radio (NR) applications, specifically in data channels, the tight demands for high data rates and excellent performance necessitate the use of LDPC codes, thanks to their capability of handling very long code words. The decoding complexity of such codewords is high specifically for applications that require high connectivity as the massive Machine Type Communications (mMTC).
Therefore, this work concentrates on a new reduced complexity LDPC decoding algorithm, the Split-Row Threshold Offset Min-Sum (SRTOMS). The algorithm has been tested under an Additive White Gaussian Noise (AWGN) channel and ensured a low computational complexity, by reducing the number of decoding iterations, maintaining basically equivalent performance compared to the original Offset Min-Sum (OMS).

Key Words: *Wireless communication, 5G New-Radio, LDPC codes, Min-Sum algorithm, MATLAB-HDL Co-Design, Error correcting codes.*

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Some Results on Elliptic Curves over Finite Fields.

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Abstract:

Let $\mathbb{F}_q$ finite field where q a power of prime number p , A elliptic curves over $\mathbb{F}_q$ is subset of projective plane off $\mathbb{F}_q$ that satisfies weierstrass equastion given by $Y^2 + a_1XY + a_3Y = X^3 + a_2X^2 + a_4X + a_6$ with a_1, a_2, a_3, a_4 and a_6 in $\mathbb{F}_q$ such that is discernment Δ is different than zero and if $p \geq 5$ the equation reduces to $Y^2 = X^3 + AX + B$ with A and B in $\mathbb{F}_q$ and $\Delta = 4A^3 + 27B^3 \neq 0$. $E(\mathbb{F}_q)$ the set of points that verifies the equation has structure of abelian group with $\mathcal{O}$ the identity element of the group is the projective point of coordinate $[0 : 1 : 0]$.
$E(\mathbb{F}_q)$ has a finite number of points t from then t verifies inequality $|q + 1 - t| \leq 2\sqrt{q}$ after give prolongement of Frobenius endomorphism ϕ_q over $E(\overline{\mathbb{F}}_q)$ is the elliptic curve defined by the same equation over $\overline{\mathbb{F}}_q$ algebraic closure of finite field $\mathbb{F}_q$ then $\phi_q : E(\overline{\mathbb{F}}_q) \to E(\overline{\mathbb{F}}_q)$ by for all (x, y) in $E(\overline{\mathbb{F}}_q)$ we have $\phi_q(x, y) = (x^p, y^p)$ then we have the identity next $\phi_q^2 + a\phi_q + p = 0$ over the elliptic curve $E(\mathbb{F}_q)$ and this morphism used to prove many other results [1],[2] ,[3] ,[4].

Key Words: *Finite field , Elliptic curves , Frobenius endomorphism .*

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Information Geometry Approach for MIMO Communication and Capacity

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Abstract:

In this work, we propose an information geometry approach (IGA) for capacity of MIMO channels. We calculate the approximation of the a posteriori information. It is formulated as an iterative m-projection process between submanifolds with different constraints. We then apply the information geometry to simplify the calculation of the m-projection since the direct calculation of the m-projection is difficult.

Key Words: *Information Geometry, MIMO channels, channel estimation.*

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

Investment of some results and properties of graphs in some algebraic structures

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Abstract:

The study of algebraic structures, using the properties of graphs, has become an exciting research topic over the last thirty years, leading to many fascinating results and questions. There are several papers on the assignment of a graph to an algebraic structure, for example see [[1], [2]]. This paper will focus on some definitions and properties of a graph associated with a certain algebraic structure and give examples of the proof of some important results.

Key Words: *Graph, Eulerian graph, Ring, Zero-divisor graph*

Introduction: .....

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Mohamed El Bachraoui

Title: N-tuple sum analogues for Ramanujan-type congruences.

Abstract: In this talk we establish supercongruence relations for truncated N-tuple sums of basic hypergeometric series. As an application, we give double, triple, and quadru- ple sum analogues of some Ramanujan-type supercongruences.

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Cyclicity of the 2-decomposed unramified Iwasawa module

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Abstract:

Let k be a real quadratic number field, and k_∞ its cyclotomic $\mathbb{Z}_2$-extension. We study the cyclicity of the Galois group X'_∞ over k_∞ of the maximal abelian unramified 2-extension, in which all 2-adic primes of k_∞ split completely. As consequence, we determinate the complete list of real quadratic number fields for which X'_∞ is cyclic.
When X'_∞ is cyclic non-trivial, we give a new infinite family of real quadratic fields, for which Greenberg's conjecture is valid.

Key Words: Iwasawa theory, 2-rank, Real quadratic fields.

Introduction: Let k be a real quadratic number field, and k_∞ its cyclotomic $\mathbb{Z}_2$-extension. We study the cyclicity of the Galois group X'_∞ over k_∞ of the maximal abelian unramified 2-extension, in which all 2-adic primes of k_∞ split completely

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Techniques in image encryption and decryption

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Abstract:

Digital image transmission over the internet need security and protection from unauthorized accesses, and stealing data. So we need encryption and decryption to protect these sensitive data.

Key Words: *Image, encryption, decryption*

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Some Characterizations of Codisk-Cyclicity for Linear Relations.

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Abstract In this paper, we introduce and study the notion of codiskcyclicity for linear relations.

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

Twisted Hessian Curve ElGamal Cryptosystem

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Abstract:

In this talk, we introduce a new public-key encryption system that is a variant of ElGamal encryption on twisted Hessian curves over the ring $\mathbb{F}_q[X]/X^2$, where $\mathbb{F}_q$ is a finite field of order $q = p^n$, p is a prime number ≥ 5, and $n \in \mathbb{N}^*$. At first, we recall the twisted Hessian curves over the ring $\mathbb{F}_q[X]/(X^2)$ and we give its essential properties, which will be used in the remainder of this work. Next, we present a new public-key cryptosystem that is a variant of the ElGamal cryptosystem on a twisted Hessian curve $H^2_{a,d}$. We conclude this work by studying its security and efficiency.

Key Words: *Twisted Hessian curve; Finite ring; Elliptic curve; ElGamal; Cryptography*

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

Codes on Dessins d'Enfants

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Abstract:

A Belyi pair consists of a compact Riemann surface and a non-constant morphism from that surface to the Riemann sphere unramified outside 0, 1 and ∞ . From a Belyi pair we construct a code by using the L-construction introduced by Goppa and the theorem of Riemann-Roch. The Grothendieck ?s correspondence between compact Riemann surface carrying maps and smouth projective algebraic curves defined over algebraic number fields is recalled. It permits to investigate algorithmic questions concerning coding theory and graph theory.

Key Words: *Goppa codes, dessins d'enfants, Riemann-Roch theorem, Belyi pair*

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

On monogenity of certain septic number fields defined by $x^7 + ax^6 + b$

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Abstract:

In this work, for every number field K generated by a root of a monic irreducible trinomial $F(x) = x^7 + a.x^6 + b \in \mathbb{Z}[x]$, we show that no odd rational prime p divides the index $i(K)$, and we give the necessary and sufficient conditions on a, b such that 2 divides $i(K)$. Specifically, we provide adequate requirements for K to be non-monogenic. Finally, several computational examples are used to illustrate our conclusions.

Key Words: *Monogeneity, Newton polygon, prime ideal factorization, Dedekind, common index divisor, Theorem of Ore*

Introduction: *Let K be a number field of degree n and $\mathfrak{o}_K$ its ring of integers; it is a free $\mathbb{Z}$-module of rank $n = [K : \mathbb{Q}]$. For any primitive element $\theta \in \mathfrak{o}_K$, let $F(x)$ be the minimal polynomial of θ, and $[\mathfrak{o}_K : \mathbb{Z}[\theta]]$ be the index of $\mathbb{Z}[\theta]$ in $\mathfrak{o}_K$. It is well known that $[\mathfrak{o}_K : \mathbb{Z}[\theta]]$ is a finite abelian group, and the index formula is given by:*

$$disc_{\mathbb{Z}}(F) = \pm[\mathfrak{o}_K : \mathbb{Z}[\theta]]^2 \cdot d_K,$$

Where d_K is the absolute discriminant of K and $disc_{\mathbb{Z}}(F)$ is the discriminant of $F(x)$. The number field K is said to be monogenic if there is some non-trivial $\beta \in \mathfrak{o}_K$, such that $(1, \beta, ..., \beta^{n-1})$ is an integral basis of K. If K has no such β we say that K is non-monogenic.
The number $i(K) = gcd\{[\mathfrak{o}_K : \mathbb{Z}[\beta]], \beta \in \mathfrak{o}_K \text{ generates } K\}$ is called the field index of K. A rational prime number p that divides $i(K)$ is called the common index prime divisor of K. If $\mathfrak{o}_K$ has a power integral basis, then the index $i(K) = 1$. Thus the field K is not monogenic, if it admits common index prime divisors. The construction of power integral bases (PIBs) or relative power integral bases (RPIBs) and testing the monogenity of number fields is a traditional problem that has been the focus of in-depth research during the last century and this century. Let $F(x) = x^7 + ax^6 + b$ be an irreducible monic trinomial and $K = \mathbb{Q}(\theta)$ be a number field generated by a complex root θ. The purpose of this work is to present necessary and sufficient conditions on a and b such that p serves as a common index divisor of K for any rational prime p. Particularly under these circumstances, K is non-monogenic. The strategy used is primarily based on Dedekind criterion, Newton's polygons and the factorization onto prime ideals.

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Dénombrement des points d'une courbe elliptique

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Abstract:

Dans cet exposé, nous présentons le théorème de Hasse ainsi que l'algorithme de Schoof, cet algorithme permet de calculer le nombre de points d'une courbe elliptique E sur un corps fini $\mathbb{F}_p$, où p un nombre premier. La complexité de cet algorithme est $\mathcal{O}\left(\log^8 p\right)$. Ensuite on montre qu'il facile de calculer l'ordre de $E\left(\mathbb{F}_q\right)$ où $q = p^n, n \in \mathbb{N}^*$, si on connait l'ordre de $E\left(\mathbb{F}_p\right)$.

Key Words: *Elliptic curve, discrete logarithm problem, Ring, Cryptography*

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

On monogeneity of nonic relative number field

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Abstract:

Let $L = K(\alpha)$ be an extension of a quadratic number field $K = \mathbb{Q}(\sqrt{d})$, where d is a square-free integer, $d \equiv 2, 3\,(mod 4)$, and α satisfies the monic irreducible polynomial $P(X) = X^9 - \beta$ belonging to $\mathfrak{o}_K[X]$ ($\mathfrak{o}_K$ is the ring of integer of K). Based on the result of Theorem **??**, we seek in this paper to simplify this result so that conditions are given only in the β-coefficients that belong to $\mathbb{Z}$. Computational examples are given to illustrate our conclusions.

Key Words: *Dedekind. Ramification, Discriminant, Power integral basis, Relative power integral basis*

Introduction: .....

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p^n -Power relatives extensions with power integral basis

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Abstract:

Let $L = K(\alpha)$ be an extension of the number field K, where α satisfies the monic irreducible polynomial $f(x) = x^{p^n} - b$ of power prime degree belonging to $\mathfrak{o}_K[X]$ and $\mathfrak{o}_K$ is the integral closure of K.
The purpose of this paper is to study the monogenity of L/K by using a new version of Dedekind's criterion, also we give an integral basis of a family of number field of degree $2p^n$ for some positive integer n. As an illustration, we get a slightly simpler computation of relative discriminant $D_{L/K}$.

Key Words: *DVR, Dedekind ring, monogenity, Relative integral basis*

Introduction: .....

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Error Correction in IoT LoRa Systems: Polar Codes for Enhanced Communication Reliability

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Abstract:

This study concentrates on the application of polar codes within LoRa modulation schemes to enhance the performance of forward error correcting codes (FEC) in Internet of Things (IoT) environments. Exploiting polar codes, recognized for their strong error correction capabilities, this work assesses their effectiveness in alleviating noise effects in LoRa-based networks, a key requirement for reliable data transmission across IoT applications. Through simulations, we analyze the performance of polar-coded versus uncoded LoRa in AWGN and Rayleigh channels, exhibiting an outstanding improvement in BER. These findings emphasize the capability of polar codes in managing the remarkable demands of long-range, low-power IoT applications, especially where LoRa is stationed.

Key Words: *IoT, LoRa, Polar Codes, Error Correction*

1 Introducing the Problem

LoRa networks, widely adopted for Internet-of-Things (IoT) connectivity, face challenges in ensuring reliable data transmission due to limitations in their error correction capabilities. The existing Hamming codes employed in LoRa, while simple, fall short in providing optimal error correction, particularly in challenging environments with low Signal-to-Noise Ratios (SNR). This directly impacts the Bit Error Rate (BER), a critical performance metric for data integrity. The significance of improving BER performance lies in enabling robust and accurate data delivery in LoRa-based IoT applications, especially those operating in harsh or noisy environments.

2 Methodology

This research focuses on investigating the potential of Polar codes as a more powerful alternative to Hamming codes for enhancing the BER performance of LoRa networks. The approach involved simulations in MATLAB, leveraging the Open-Source LoRa framework:

- ***Replacing Hamming Codes with Polar Codes:*** *The existing Hamming codes in the LoRa physical layer were substituted with Polar codes. This involved selecting a suitable Polar code construction and defining the information and frozen bits for encoding.*
- ***Evaluating BER Performance:*** *The primary focus of the simulation was to assess the BER performance of LoRa with Polar codes across a range of Eb/N0 values. This allowed for a direct comparison with the BER performance of the existing Hamming codes.*
- ***Addressing LoRa-Specific Challenges:*** *Techniques such as spectrum-based LLR extraction and symbol-aware enhancements could be applied to address LoRa-specific challenges.*

3 Key Results

The simulation results demonstrate that Polar codes offer significant improvements in BER performance for LoRa networks:

- *At a given Eb/N0, LoRa with Polar codes achieved a lower BER compared to LoRa with Hamming codes.*
- *A lower BER translates to improved signal reception and decoding reliability, potentially extending the effective communication range of LoRa networks.*

4 Implications and Applications

The enhanced BER performance offered by Polar codes has important implications for LoRa-based IoT applications:

- ***Increased Data Reliability:*** *A lower BER ensures robust data delivery.*
- ***Extended Communication Range:*** *Polar codes can improve signal robustness, especially in low-SNR conditions.*

5 Conclusion

The integration of Polar codes into LoRa modulation schemes offers a promising pathway to overcoming existing challenges in IoT communication systems. Our study demonstrated that replacing Hamming codes with Polar codes significantly enhances the Bit Error Rate (BER) performance in LoRa networks under both AWGN and Rayleigh fading conditions. These improvements directly address the growing demand for reliable and long-range communication in IoT applications.
Future work includes exploring hardware implementation to validate the practical feasibility of deploying Polar codes in LoRa devices. Additionally, applying this approach to more complex channel models, such as those simulating urban environments, will provide a more comprehensive understanding of its impact on IoT networks.

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5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

Double cyclic Codes over $\aleph = \mathbb{Z}_2 + \zeta\mathbb{Z}_2$

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Abstract:

This paper studies the structure and properties of cyclic codes and their generalization to double-cyclic codes over $\aleph = \mathbb{Z}_2 + \zeta\mathbb{Z}_2$. Specifically, we focus on determining generator polynomials for these double-cyclic codes. Our approach relies on algebraic techniques to derive the generator polynomials, thus providing a comprehensive framework for their construction. In addition, we explore the ranks and minimal covering sets associated with double-cyclic codes. Through rigorous analysis, we establish the conditions under which these codes exhibit optimal properties.

Key Words: *Cyclic codes, Double-cyclic codes*

Introduction: *Cyclic codes are a fundamental class of linear codes widely used in error detection and correction within digital communication and data storage systems [1, 2, 3, 5, 6]. This type of code can also intersect with cryptography, as noted in the references [4, 7]. In some instances, these codes are employed to implement secure cryptographic protocols, while in other cases, they are utilized in attacks against cryptographic systems.*

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Characterizing Subgroups of Class Groups in Biquadratic Fields

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Abstract:

This study investigates the properties of class groups associated with the biquadratic fields $Q(i, \sqrt{d})$, where $i = \sqrt{-1}$ and d is a square-free integer. Our objetif is to identify conditions under which these class groups contain subgroups isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$, corresponding to a 2-rank of at least 3.

Key Words: *Class group, class number, Biquadratic fields, 2-rank*

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Advancing the Construction of Quantum Codes over Non-Chain Rings: A Generalized Approach

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Abstract:

This paper constructs quantum error-correcting codes (QECCs) and entanglement-assisted quantum error-correcting codes (EAQECCs) through the Euclidean hull and the sum of cyclic codes over non-chain rings $\mathcal{R}$. The given approach extends our prior work [1] to non-chain rings, specifically $\mathbb{F}_p[\mathbf{X}]/(\mathbf{X}^l - \mathbf{X}^{l-1})$, where p is an odd prime and $l \geq 2$. We investigate generator polynomials, code dimensions, and derive new QECCs and EAQECCs with enhanced parameters, such as increased minimum distance or larger dimensions. Additionally, we define the isometric Gray map to adapt these codes for quantum applications, advancing the efficacy of quantum error correction methods.

Key Words: *Cyclic codes, Gray map, (entanglement-assisted) quantum error-correcting codes*

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Dynamical Systems and Biomathematics

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TITRE
Distributions Stationnaires dans les Modèles Épidémiques Stochastiques : Une Approche Ergodique

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Abstract:

Cet article explore l'application de la théorie ergodique pour comprendre les modèles épidémiques stochastiques, en se concentrant sur leur comportement à long terme sous influence aléatoire. La théorie permet d'analyser si ces modèles atteignent une distribution stationnaire, ce qui est essentiel pour identifier des comportements durables dans la dynamique épidémique. Un cadre épidémique stochastique est présenté, intégrant des taux de natalité logistique et un taux d'incidence saturé, afin de mieux comprendre la dynamique de certaines maladies infectieuses. Ces maladies constituent une menace pour la santé humaine et le développement durable, impactant divers aspects de la vie quotidienne et freinant le progrès socio-économique à l'échelle mondiale.
L'étude mathématique commence par la démonstration de l'existence et de l'unicité d'une solution positive globale pour le modèle proposé. Elle aborde ensuite les conditions dans lesquelles l'infection peut persister ou devenir extincte. De plus, les conditions nécessaires à l'existence et à l'unicité des distributions stationnaires ergodiques parmi les solutions du modèle sont établies grâce au développement d'une fonction de Lyapunov stochastique appropriée. Les résultats théoriques sont validés par des simulations numériques.

Key Words: *Modèle épidémique stochastique · Distribution stationnaire · Ergodicité · Extinction · Persistance*

Introduction: *Plusieurs études épidémiologiques visent à identifier les facteurs de risque et les déterminants des états et événements liés à la santé, conduisant à des infections dans des populations spécifiques. Les recherches sur les maladies infectieuses ont mis en évidence des concepts épidémiologiques tels que la période d'incubation et la résistance, en les appliquant également aux maladies non infectieuses.*
En général, les maladies se classent selon leurs caractéristiques cliniques ou micro-biologiques, mais définir une nouvelle maladie commence par une caractérisation épidémiologique. Pour les professionnels de santé publique, ces caractéristiques épidémiologiques sont essentielles dans l'élaboration d'un programme de contrôle. La collaboration entre cliniciens, microbiologistes et épidémiologistes est indispensable pour maîtriser, traiter et prévenir la propagation d'une épidémie. Cependant, chaque spécialiste apporte une perspective et une contribution spécifiques dans ce domaine [1]. La théorie ergodique est essentielle dans les modèles épidémiques stochastiques, car elle permet d'étudier les comportements à long terme des épidémies malgré les fluctuations aléatoires. En assurant la convergence des moyennes temporelles vers les moyennes d'ensemble, elle facilite la prévision des proportions stables de populations infectées, rétablies ou sensibles. Cela permet aux responsables de la santé publique de concevoir des stratégies de contrôle fondées sur une compréhension approfondie des dynamiques épidémiques moyennes, tout en prenant en compte les incertitudes du système.
Il est crucial de reconnaître le rôle que peut jouer la fonction de densité de probabilité dans un modèle stochastique. En effet, déterminer cette fonction permet de mieux comprendre les propriétés dynamiques d'un système stochastique en termes de signification statistique. La fonction de densité de probabilité associée à une distribution stationnaire satisfait l'équation de Fokker-Planck, bien que sa résolution soit complexe, surtout pour les modèles stochastiques de haute dimension [2].

De nombreux chercheurs ont choisi d'utiliser le modèle de croissance logistique. Par exemple, Juan Liu, Rajasekar et Pitchaimani [3, 4] ont proposé un modèle SIRS intégrant une fonction de taux d'incidence non linéaire et une croissance logistique. Sur la base de la discussion précédente, cet article vise à examiner le comportement dynamique d'un modèle épidémique stochastique, en mettant particulièrement l'accent sur les concepts de la théorie ergodique.
L'analyse de la dynamique d'un modèle épidémique stochastique à travers le prisme de la théorie ergodique permet de mettre en lumière des comportements émergents, tels que les différentes phases épidémiques, les périodes de rémission et les transitions entre divers états épidémiques[5]. Ces résultats peuvent fournir des informations essentielles pour la planification et l'application de stratégies de contrôle des épidémies.

En résumé, cet article vise à intégrer l'analyse dynamique des modèles épidémiques stochastiques avec les concepts de la théorie ergodique afin de mieux comprendre la complexité des phénomènes épidémiques et de proposer des pistes pour leur gestion

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Hopf bifurcation in a delayed fractional-order prey-predator model with Holling type II functional response, incorporating a refuge area for prey in the presence of toxicity

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Abstract:

Modeling population systems with time delay and fractional order has recently led to more realistic representations. The inclusion of fractional order enhances model behavior, while time delay contributes to richer stability results. In this paper, we introduce a fractional-order delayed prey-predator dynamic system across two zones: a reserved area and an open-access fishery zone. Here, predator species consume prey in both the reserved and open-access zones, with predation in each zone following a Holling type II functional response. A time delay is incorporated to account for the period required to initiate an attack. This study aims to explore the dynamics of the model using two approaches: an integer-order differential equation model and a fractional-order differential model. Both systems are analyzed through bifurcation analysis and numerical simulations, examining the occurrence of Hopf bifurcation with delay as the bifurcation parameter. Our findings indicate that system stability is significantly influenced by the fractional order and the delay, whereas the integer-order model is primarily affected by delay alone. Thus, fractional-order modeling provides a more accurate and comprehensive understanding of the system's dynamics.

Key Words: *Prey-predator model, toxicity, time delay, stability, Hopf bifurcation.*

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CNN-Based Facial Emotion Recognition: A Comprehensive Survey of Recent Advances and Challenges

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Abstract:

Facial Emotion Recognition (FER) has emerged as a prominent area within computer vision, driven by its wide-ranging applications in sectors such as education, medicine, security, and robotics. While prior reviews have examined various machine learning and deep learning approaches to FER, this paper specifically focuses on Convolutional Neural Networks (CNNs). In recent years, CNN-based models have been introduced as effective solutions to address the challenges inherent in FER. This comprehensive review aims to identify, analyze, and evaluate the current state of CNN-based methods for facial emotion recognition. We have systematically reviewed seminal studies published between 2016 and 2023 from leading databases, including Scopus, IEEE, Web of Science, and Springer. Our findings suggest that CNNs have been widely adopted in FER research, yielding promising results and demonstrating significant potential to enhance the accuracy of emotion recognition from facial expressions.

Key Words: *Convolutional Neural Network (CNN), Facial Emotion Recognition (FER), Systematic Review, Deep Learning.*

Introduction: *Nonverbal communication plays an essential role in expressing emotional states [1], encompassing behaviors such as facial expressions, vocal intonation, and body posture. Among these, facial expressions hold particular significance for emotion detection. Facial Emotion Recognition (FER) involves the analysis of facial images or videos to identify micro-expressions, which are used to infer an individual's emotional state. This process typically includes two key stages: feature extraction and classification, which together present considerable challenges within the field of computer vision. FER has a broad range of applications across diverse domains, including human-computer interaction, education, healthcare, and security, rendering it a rich and dynamic area of research. Ekman and Friesen [2] classified seven primary emotions based on facial expressions-anger, disgust, fear, happiness, sadness, surprise, and neutral-demonstrating the universality of these emotional expressions. The majority of research in FER continues to focus on these core emotional categories.*

The rapid development of Machine Learning (ML) has given rise to various methods for FER, including Support Vector Machines (SVM) [3], Bayesian Networks [4], and descriptor-based classifiers. However, these traditional techniques have demonstrated limitations in terms of accuracy and robustness. In contrast, the emergence of Deep Learning, particularly Convolutional Neural Networks (CNNs), has provided a powerful means to overcome these challenges. CNNs [5][6] are among the most widely used neural network architectures, especially effective in supervised and unsupervised learning tasks involving large-scale datasets. Their proven success in image classification and recognition tasks has translated into promising applications in FER. The primary advantages of CNNs, such

as parameter sharing, sparsity of connections, and invariance to geometric transformations [7], make them particularly well-suited for FER tasks, offering enhanced speed and accuracy.
Although numerous review articles on FER have been published between 2016 and 2023, focusing on both Machine Learning and Deep Learning approaches, many do not specifically address CNN-based models. Notably, Laith Alzubaidi et al. [8] offer a comprehensive survey of essential Deep Learning techniques, including CNNs, and review popular CNN architectures such as AlexNet, GoogLeNet, and ResNet. Similarly, works by Yan Wang et al. [9] and Gilderlane Alexandre et al. [10] focus on multimodal affective computing and 3D FER methods, respectively. However, these studies do not offer an exclusive focus on CNNs for FER, which is the primary objective of this paper.
This systematic review seeks to bridge this gap by offering an in-depth analysis of current CNN-based approaches to FER. By critically examining the relevant literature, this work provides an essential resource for advancing the understanding of CNN-based facial emotion recognition and identifies potential directions for future research. The objective of this paper is to fill the gap in the existing literature by providing an in-depth analysis of current CNN-based approaches to Facial Emotion Recognition. By critically examining the relevant studies, this review aims to provide an essential resource for enhancing the understanding of CNN models applied to facial emotion recognition, while identifying remaining challenges and suggesting directions for future research.

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Stabilization of bilinear systems with distributed delays using the Banach state space decomposition method

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Abstract:

The strong stabilization of the considered system is established by presenting sufficient conditions based on an observability condition expressed by the semigroup solution of the linear part of the bilinear system. An explicit estimate on the convergence of the decay rate is established. Moreover, some sufficient conditions are used to discuss the weak stabilization of the considered system. Additionally, an illustrative example with numerical simulations is included.

Key Words: Distributed delay; strong stabilization; Banach state space; decay estimate; bilinear systems

Introduction: The study of bilinear systems is an active area of the theory of nonlinear systems and has numerous applications in various fields of applied mathematics. Their nonlinearity makes their analysis challenging. However, there are several applications, such as modeling population dynamics, chemical reactions, engineering, and control systems. We can find more information on this subject in the references [1, 2]. It is necessary to use both analytical and numerical methods when analyzing bilinear systems. To derive stability conditions, analytical methods use mathematical tools such as semigroup theory, Lyapunov stability theory, and Laplace transforms. To study the behavior of these systems under different conditions, numerical methods use computer simulations. For instance, we refer to the following references [3, 4].

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Some Mathematical models based on industry 4.0 for healthcare

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Abstract:

Our work focuses on some mathematical models and their applications in medical Industry 4.0. More precisely, it involves investing in the mathematical models of digital twins, to study, experiment, adapt and improve medical interventions. The work clarifies the importance of Digital Twins DT in health care services and describes the technical process used in DT for industry 4.0. Besides, basing on the mathematical models, we give the supportive features of DT that create a virtual model which is a virtual copy of the physical model in order to identify some related applications in healthcare. We, then, discuss the advantages and challenges related to DT as a technical solution. In addition, the findings indicate that there is a significant relationship between DT capabilities as a service and mass individualization.

Key Words: *: Digital twin; Industry 4.0; Digital health; IoT and AI; Data maths model; Optimization and Simulation.*

Introduction: *The Digital Twins of a physical system is an adaptive computer analog which exists in the cloud and adapts to changes in the physical system dynamically. Digital Twins (DT) has emerged as one of the most active components in smart manufacturing, garnering significant attention from enterprises, research institutes, and researchers. By creating a Digital Twins, manufacturers can simulate different scenarios and test various configurations without disrupting the actual production process. This allows for more efficient testing and optimization of production processes, as well as improved quality control and predictive maintenance. Moreover, A Digital Twin's main advantage is that it gives real time data that can aid in learning, reasoning, and understanding how objects and systems functionned. It enables users to analyze, model, and optimize a physical objects performance across its lifespan. In this paper, after reviewing the literature on the subject, we presented a framework of the DT in smart manufacturing in health sector, which includes optimization, predictive Maintenance, quality Control, design, and simulation, which can be a good guide for future studies.*
This paper provides a comprehensive overview of DT solutions for healthcare. We delve into the diverse spectrum of Digital Twins, delve into the different parameters they can measure, explore their myriad potential applications within industry 4.0. in this context, the virtual model should incorporate changes in the physical model communicated through sensor data. the virtual model typically exists as a computer program in the cloud, aspects of cloud computing become important along with the placement of the physical model in the context of the sensors as a part of the IoT. The objective of the modeling and simulation phase of the digital twin is to create a virtual model which is a mirror image of the physical model. On this basis, some possible paradigms and mathematical models for further research activities will be discussed.

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Stability analysis of a predator-prey eco-epidemic model with hunting cooperation and fear effects

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Abstract:

We propose a system of delay differential equations to represent predator-prey eco-epidemic dynamics. The model incorporates the effects of predation risk-induced fear on prey and collaborative hunting strategies of predators. The crucial aim of the study is to analyze the system's dynamics under the influence of predator-hunting cooperation, discrete-time delay, and fear effect. We examine key mathematical characteristics of the proposed model, including boundedness, persistence, local stability, and Hopf bifurcation near the system's positive steady state. Additionally, we conduct numerous numerical simulations with different parameter sets to validate our analytical results.

Key Words: *eco-epidemiological model, discrete-time delay, hunting cooperation, Hopf-bifurcation, fear effect.*

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Nonlinear Analysis and Applications

COMMON PROPERTIES OF THE BOUNDED RIGHT LINEAR OPERATORS AB and BA IN THE QUATERNIONIC SETTING.

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Abstract. For a bounded right linear operators A and B in a two-sided quaternionic Banach space X, we study the common spectral properties for the bounded right linear operators AB and BA. Furthermore, we investigate some relations betwen the bounded right linear operators $Q_q(AB)$ and $Q_q(BA)$, where $Q_q(T) := T^2 - 2\,\mathrm{Re}(q)T + |q|^2 I$, for all non-zero quaternion q and all bounded opertor T.

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Multiplicity results for Kirchhoff-double phase problem with nonlinear boundary condition

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Abstract:

The purpose of this talk is to study the multiplicity of nontrivial weak solutions for the following Kirchhoff-double phase problem:

$$\begin{cases} m\left(L(u)\right) D(u) = |u|^{p(x)-2}u + b(x)|u|^{q(x)-2}u & \text{in } \Omega, \\ m\left(L(u)\right)\left(|\nabla u|^{p(x)-2}u + b(x)|\nabla u|^{q(x)-2}u\right)\cdot \nu = \lambda g(x,u) \text{ on } \partial\Omega, \end{cases}$$

where $L(u) = \int_\Omega (\frac{1}{p(x)}|\nabla u|^{p(x)} + \frac{b(x)}{q(x)}|\nabla u|^{q(x)})dx$ and D is the double phase operator with variable exponents. The goal is to determine the precise positive interval of λ for which the above problem admits at least two nontrivial weak solutions without assuming the Ambrosetti-Rabinowitz condition. Next, we give a result on the existence of an unbounded sequence of nontrivial weak solutions by employing the Fountain Theoreom with Cerami condition.

Key Words: *Double phase problems, Musielak-Orlicz space, critical point theory, Cerami condition*

Introduction: *The investigation of solutions of superlinear problems governed by the double phase operator is a novel and significant area of research, since offers insights into various applications in the field of mathematical physics, including elasticity theory, strongly anisotropic materials, Lavrentiev's phenomenon.*

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Existence of solutions for p-Laplacian parabolic equation

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Abstract: The main objective of this communication is to study a weak solutions for the following parabolic problem:

$$\begin{cases} u_t - \Delta_p u + |u|^{p-2}u = 0 & \text{in } \Omega,\ t > 0, \\ \sigma u_t + |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} = g(u) & \text{on } \partial\Omega,\ t > 0, \\ u(x;0) = u_0(x) & \text{in } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is an open bounded domain with smooth boundary $\partial\Omega$. By using the Galerkin approximation and a family of potential wells, we obtain the existence of global solution and finite time blow-up under some suitable conditions. On the other hand, the results for asymptotic behavior for certain solution with positive initial energy are also given.

Keywords: Parabolic problem; Dynamical boundary condition; Blow-up

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SYSTEM OF FUNCTIONAL EQUATIONS ON SEMIGROUPS

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Abstract:

Our aim is to solve a system of functional equations closely related to trigonometric functional equations. This allows us to express quaternion-valued multiplicative functions in terms of complex-valued multiplicative functions. As an application of our results, we give the continuous quaternion-valued solutions of a functional equation on $(\mathbb{R}, \cdot)$.

Key Words: *Functional equation, multiplicative function, quaternion, semigroup.*

Introduction:

Throughout this paper we let S be a semigroup (i.e., a non-empty set equipped with an associative composition rule $(x, y) \mapsto xy$). A functional equation is an equation in which the unknown or unknowns are functions. Solving a functional equation is expressing the unknown function or functions in terms of simple functions, for example, in terms of multiplicative functions. Recall that a function χ from S into a skew field is said to be multiplicative if $\chi(xy) = \chi(x)\chi(y)$ for all $x, y \in S$. A function f on S is said to be central if $f(xy) = f(yx)$ for all $x, y \in S$. For any function $f : S \to \mathbb{C}$, we denote by $\Re(f)$ its real part, $\Im(f)$ is its imaginary part, and $\overline{f}$ is its complex conjugate. Our main goal is to express quaternion-valued central multiplicative functions in terms of complex-valued multiplicative functions. To do this, we shall first solve the following system of functional equations

$$\begin{cases} a(xy) = a(x)a(y) - b(x)\overline{b}(y) \\ b(xy) = a(x)b(y) + b(x)\overline{a}(y) \end{cases}, \tag{1}$$

where $a, b : S \to \mathbb{C}$ are unknown functions and a is central. The system (1) is closely connected to the trigonometrical functions. Indeed, for any $\theta \in \mathbb{R}$, the pair a, b defined by

$$a(x) = \cos(x),\ b(x) = \sin(x)e^{i\theta}, \quad x \in \mathbb{R},$$

is a solution of the system (1) on the group $(\mathbb{R}, +)$.

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Multiple homoclinic solutions for a class of discrete fractional p-Laplacian equations

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Abstract:

In this presentation, we study by the variational approach the multiplicity of homoclinic solutions for the next discrete fractional p-Laplacian equation

$$\begin{cases} (-\Delta_{\mathcal{G}})_p^s u(\xi) + V(\xi)|u(\xi)|^{p-2}u(\xi) = f(\xi, u(\xi)) + \lambda h(\xi)|u(\xi)|^{q-2}u(\xi) & \text{for } \xi \in \mathbb{Z}, \\ u(\xi) \longrightarrow 0 & \text{as } |\xi| \longrightarrow \infty \end{cases}$$

where $s \in (0,1)$, $2 < r < p < q$, $V : \mathbb{Z} \longmapsto \mathbb{R}^+$, λ is a positive parameter, $h \in C(\mathbb{Z}, \mathbb{R})$, and $f \in C(\mathbb{Z} \times \mathbb{R}, \mathbb{R})$. Upon appropriate assumptions on the potential V and nonlinearity f, we use the Nehari manifold in conjunction with fibering maps to prove the multiple homoclinic solutions for the discrete fractional p-Laplacian equation.

Key Words: Discrete fractional p-Laplacian, Homoclinic solutions, Nehari manifold, Fibering maps.

Introduction: The discrete fractional p-Laplacian operator is a mathematical operator that generalizes the classical p-Laplacian operator to accommodate fractional orders of differentiation in discrete settings. This operator is particularly useful in various fields, such as image processing, anomalous diffusion, optimization, game theory, quantum mechanics, finance, etc. For more information, see [1, 2, 3, 4] and references therein.

Recently, several researchers have innovatively utilized variational methods to investigate problems concerning the discrete fractional p-Laplacian operator (for $p = 2$ or $p > 2$), as documented in [5, 6, 7, 8, 9]. The variational method proves effective in addressing discrete fractional problems, particularly concerning the existence and multiplicity of solutions. At the same time, we note that the first study of discrete fractional problem by using Nehari manifold method was considered by Ju et al. in [8], in which they investigated the following discrete fractional p-Laplacian equation

$$\begin{cases} (-\Delta_{\mathcal{G}})_p^s u(\xi) + V(\xi)|u(\xi)|^{p-2}u(\xi) = \lambda\varrho_1(\xi)|u(\xi)|^{q-2}u(\xi) + \varrho_2(\xi)|u(\xi)|^{r-2}u(\xi) & \text{for } \xi \in \mathbb{Z}, \\ u(\xi) \longrightarrow 0 & \text{as } |\xi| \longrightarrow \infty \end{cases} \tag{1}$$

where $V : \mathbb{Z} \longrightarrow (0, \infty)$, $\lambda > 0$, $1 < q < p < r < \infty$, $\varrho_1 \in l^{\frac{p}{p-q}}$, and $\varrho_2 \in l^{\infty}$. Under appropriate assumptions, they found two distinct nontrivial and nonnegative homoclinic solutions for Eq. (1) by using the link between the Nehari manifold and fibrering maps.

In this presentation, we outline our results in [9], where we extend the conclusions of [8] by considering a more general form of the term $f(\xi, u)$ in the equation (1), beyond the specific case $f(\xi, u) = \varrho_2(\xi)|u(\xi)|^{r-2}u(\xi)$. Specifically, we investigate the problem

$$\begin{cases} (-\Delta_{\mathcal{G}})_p^s u(\xi) + V(\xi)|u(\xi)|^{p-2}u(\xi) = f(\xi, u(\xi)) + \lambda h(\xi)|u(\xi)|^{q-2}u(\xi) & \text{for } \xi \in \mathbb{Z}, \\ u(\xi) \longrightarrow 0 & \text{as } |\xi| \longrightarrow \infty \end{cases} \tag{2}$$

where $0 < s < 1 < p < \infty$ are constants, λ is a parameter, $V : \mathbb{Z} \longmapsto \mathbb{R}^+$ denotes a potential function, and the nonlinearity $f(\xi, .) \in C(\mathbb{R}, \mathbb{R})$ for all $\xi \in \mathbb{Z}$. Thus, by using the Nehari manifold combined with fibering maps, we establish the multiplicity of nontrivial homoclinic solutions for Eq. (2).

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Existence of solutions for a Steklov eigenvalue problem

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Abstract:

In this work, we study a class of (p, q) elliptic problems under Steklov-type boundary conditions in a smooth bounded domain Ω in $\mathbb{R}^N$. Using variational methods, we establish the existence of a continuous and unbounded set of positive generalized eigenvalues.

Key Words: *Steklov eigenvalue problem, (p, q)-Laplacian, nonlinear boundary conditions, mountain pass theorem.*

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Existence and non existence of solutions for a bi-nonlocal $p(x)$-Kirchhoff equation

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Abstract:

This paper interested the existence and non existence of solutions for bi-nonlocal $p(x)$-Kirchhoff equation. By sufficient conditions and technical approach is based on variational method, existence and multiplicity results of the solutions are established

Key Words: *Bi-nonlocal elliptic problem ;p(x)-Laplacian operator; Variational method; ; Variational method; Fountain theorem; ;(PS) condition.*

Introduction: *In this paper, we treat the question of the existence and non existence of weak solutions for the following bi-nonlocal $p(x)$-Kirchhoff equation,*

$$\begin{cases} -M\left(\int_\Omega \frac{1}{p(x)}|\nabla u|^{p(x)}dx\right)\Delta_{p(x)}u = \lambda f(x,u)\left[\int_\Omega F(x,u)dx\right]^r + \mu g(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ $(N > 1)$ is bounded smooth domain, $f, g : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ and $M : \mathbb{R}^+ \to \mathbb{R}^+$ are continuous functions satisfying conditions which will be stated later. $F(x,u) = \int_0^u f(x,s)ds$, $\lambda, \mu \in \mathbb{R}$, $r > 0$ is real parameter, $p \in C(\overline{\Omega})$ with $N > p(x) > 1$ and $\Delta_{p(x)}u = div(|\nabla u|^{p(x)-2}\nabla u) = \sum_{i=1}^{N}\left(|\nabla u|^{p(x)-2}\frac{\partial u}{\partial x_i}\right)$ is the $p(x)$-Laplacian operator.
.

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Étude de la multiplicité des solutions faibles d'un problème singulier $(p(x), q(x))$-biharmonique avec conditions de non-flux

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Mots-clefs : Méthodes variationnelles, problème singulier, opérateur p(.)-biharmonique, solutions faibles.

Résumé. *Dans ce travail, nous nous sommes concentrés sur l'existence de trois solutions pour l'opérateur $(p(\cdot), q(\cdot))$-biharmonique avec un terme de Hardy à exposant variable, sous des conditions aux bords de non-flux. Nous avons montré, à l'aide du théorème de Ricceri, que le problème admet au moins une solution faible non triviale. De plus, en utilisant deux théorèmes de Bonanno, nous avons démontré que le problème admet au moins trois solutions distinctes.*

1 Introduction

Dans ce travail on s'interesse à l'existence et à la multiplicitè des solutions faibles du problème suivant:

$$(P_\lambda)\begin{cases} \Delta^2_{p(x)}u + \Delta^2_{q(x)}u + a(x)|u|^{p(x)-2}u = \mu m(x)\dfrac{|u|^{s(x)-2}u}{|x|^{s(x)}} + \lambda f(x,u) & \text{dans } \Omega \\ u = constante,\ \Delta u = 0 & \text{sur } \partial\Omega, \\ \int_{\partial\Omega}\frac{\partial}{\partial n}\Big(|\Delta u|^{p(x)-2}\Delta u\Big) + \frac{\partial}{\partial n}\Big(|\Delta u|^{q(x)-2}\Delta u\Big)ds = 0. \end{cases}$$

où $\Omega \subset \mathbb{R}^N (N > 2)$ est un domaine régulier et borné avec $0 \in \Omega$, λ est un paramètre positif réel, les fonctions $p(x), q(x), r(x) \in C(\overline{\Omega})$ et $f : \Omega \times \mathbb{R} \to \mathbb{R}$

est une fonction de Carathéodory verfiant certaines conditions qui seront citées ci-dessous.
Nous supposons les conditions suivantes
$H(r,q,p)$ $\quad 1 < q^- < q^+ < p^- < p^+ < \frac{N}{2}$.
(s) $p^+ < s^- \leq s(x) < \min\{N, p_2^\star(s)\}$ pour tout $x \in \overline{\Omega}$, et $s^+ - \frac{1}{2} < s^-$, où

$$p_2^\star(s) = \frac{N - s(x)}{N} p_2^\star(x) \quad \text{pour tout} \quad x \in \overline{\Omega}.$$

(a) $a \in L^\infty(\Omega)$ et il est existé $a_0 > 0$ tel que $a(x) \geq a_0$ pour tout $x \in \Omega$.
(m) $m \in L^{\gamma(x)}(\Omega)$ est une fonction changeant de signe, où $\gamma \in C_+(\overline{\Omega})$ et $\frac{1}{p_2^\star(x)} + \frac{1}{\gamma(x)} < \frac{1}{s(x)}$ pour tout $x \in \overline{\Omega}$.
(f_1)

$$|f(x,t)| \leq a_1 + a_2|t|^{z(x)-1} \text{ pour tout } (x,t) \in \Omega \times \mathbb{R},$$

avec $a_1,\ a_2 > 0$ et $1 < z(x) < p^-$, $\forall x \in \overline{\Omega}$.
(f_2) Il existe $\nu(x)$ tel que $p^+ < \nu^- < \nu(x) < p_2^\star(x)$, et

$$\lim_{s\to 0} \sup_{x\in\Omega} \frac{F(x,s)}{|s|^{\nu(x)}} < +\infty$$

.

Sous les conditions (s), (a), (m), (f_1) et (f_2), nous établissons l'existence de trois solutions faibles pour le problème (P_λ).

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On the problem of unique continuation for a nonlinear elliptic equation

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Abstract:

We consider a nonlinear problem driven by the p−Laplacian differential,

$$-\Delta_p u + V|u|^{p-2}u = 0,$$

where $1 < p < \infty$ and V is a bounded function. Using the Almgren frequency function, we show the unique continuation principle, for the following different solutions of the problem.

Key Words: *Unique continuation; Almgren frequency; p-Laplacian operator.*

Introduction: *This study establishes a unique continuation principle for solutions to the following nonlinear elliptic problem:*

$$\Delta_p u + V|u|^{p-2}u = 0,$$

in an open and connected set $G \subseteq \mathbb{R}^n$ with $n \geq 2$ and $2 \leq p < \infty$. Here, $\Delta_p u := \nabla \cdot (|\nabla u|^{p-2}\nabla u)$ represents the p-Laplacian differential operator. The function V satisfies:

$$|V(x)| < M \quad a.e.\ in\ G.$$

If u from this equation vanishes on a small open subset $\omega \subset G$, then it must also vanish throughout G. This unique continuation property is a crucial aspect for understanding the behavior of solutions to such nonlinear problems.

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On the Characterization of the Spectrum of a Fourth-Order Eigenvalue Problem

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Abstract:

In this paper we aim to study the spectrum of the p biharmonic operator under suitable boundary conditions in a bounded domain. More precisely we study the fourth order equation

$$\Delta_p^2 u + 2\xi.\nabla(|\Delta u|^{p-2}\Delta u) + |\xi|^2|\Delta u|^{p-2}\Delta u = \mu m|u|^{p-2}u,$$

in a bounded smooth domain Ω of $\mathbb{R}^N (N \geq 1)$. Where $\xi \in \mathbb{R}^N, \mu \in \mathbb{R}, \Delta_p^2$ denotes the p-biharmonic operator defined by $\Delta_p^2 u = \Delta\left(|\Delta u|^{p-2}\Delta u\right)$ and $m \in \{m \in L^\infty(\Omega)/ \text{ meas } \{x \in \Omega / m(x) > 0\} \neq 0\}$.

Key Words: *p-biharmonic operator, Nonlinear spectral theory, third order spectrum*

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Existence of solutions for a discrete problem with mixed boundary conditions

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Abstract:

In this paper, we consider a nonlinear discrete problem originated from a capillary phenomena, involving the $p(k)$-Laplacian-like operators with mixed boundary condition. Under appropriate assumptions on the function f and its primitive F near zero and infinity, we investigate the existence and multiplicity of nontrivial solutions by using variational methods and critical point theory.

Key Words: *Critical point theory, discrete problems, variational methods, $p(k)$-Laplacian-like operators*

Introduction: *The aim of this article is to establish the existence and multiplicity of solutions for a general discrete problem originated from a capillary phenomena, involving the $p(k)$-Laplacian-like operators with mixed boundary condition of the following form:*

$$(P)\ \begin{cases} -\Delta\big(m(r-1)a(r,\Delta u(r-1))\big) + \beta(r)|u(r)|^{p(r)-2}u(r) = f(r,u(r)),\ r \in [1,N]_{\mathbb{N}}, \\ u(0) = \Delta u(N) = 0, \end{cases}$$

where $a(.,.)$ is defined as follows:

$$a(r,s) = \left(1 + \phi_c\left(|s|^{p(r-1)}\right)\right)|s|^{p(r-1)-2}s,\ \text{for all } r \in [1,N]_{\mathbb{N}} \text{ and } s \in \mathbb{R},$$

and ϕ_c is the so-called mean curvature operator defined as

$$\phi_c(s) := \frac{s}{\sqrt{1+s^2}},\ s \in \mathbb{R}.$$

Let $[1,N]_{\mathbb{N}}$ be the discrete interval given by $[1,N]_{\mathbb{N}} := \{1,2,\ldots,N\}$, where $N \geq 2$ is a positive integer and Δ denotes the forward difference operator $\Delta u(r) := u(r+1) - u(r)$. In addition, $m : [1,N+1]_{\mathbb{N}} \longrightarrow [1,+\infty)$, $\beta : [0,N]_{\mathbb{N}} \longrightarrow [1,+\infty)$ and $p : [0,N+1]_{\mathbb{N}} \to [2,+\infty)$ are given functions, and for every fixed $r \in [0,N]_{\mathbb{N}}$, $f(r,.) : \mathbb{R} \to \mathbb{R}$ is a continuous function that checks some conditions.

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INDEPENDANCE ET STABILITE DE QUELQUES EQUATIONS FONCTIONNELLES

MOHAMED TIAL

Résumé

Soit S un semi-groupe (non nécessairement abélien) et σ_1, σ_2 sont deux automorphismes involutifs sur S. On étudie le problème de la stabilité et le phénomène d'indépendance de quelques équations fonctionnelles. Tout d'abord, nous allons prouver que les deux inégalités

$$|f(xy) + f(x\sigma_1(y)) - 2f(x)| \leq \varepsilon,\ x, y \in S, \tag{0.1}$$

et

$$|g(xy) - g(x)g(y)| \leq \varepsilon,\ x, y \in S, \tag{0.2}$$

d'une part, et d'autre part on montre que les inégalités (0.1) et

$$|g(xy) + g(x\sigma_2(y)) - 2g(x) - 2g(y)| \leq \varepsilon,\ x, y \in S, \tag{0.3}$$

sont indépendantes sous la condition que g est centrale i.e. $g(xy) = g(yx)$ pour tout $x, y \in S$. Comme application, nous donnons un résultat similaire sur la stabilité et l'indépendance de l'équation additive de Cauchy et l'équation fonctionnelle de Jensen, on montre que les inégalités (0.1) et

$$|g(xy) - g(x) - g(y)| \leq \varepsilon, \quad x, y \in S, \tag{0.4}$$

sont étrangères. Deuxièmement, nous étudierons les problèmes de stabilité et d'indépendance de l'équation fonctionnelle de Jensen et de Drygas, i.e. l'indépendance des inégalités (0.1) et

$$|g(xy) + g(x\sigma(y)) - 2g(x) - g(y) - g(\sigma(y))| \leq \varepsilon,\ x, y \in S. \tag{0.5}$$

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Key words and phrases. Equations fonctionnelles, Indépendance, Stabilité.

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Numerical Analysis and Scientific Computing

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Bernstein polynomials and local polynomial for the smooth estimation of regression

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Abstract:

This study presents a new estimator of regression in Bernstein basis and local polynomial kernel, via polar forms. It was written in the form

$$\sum_{i=1}^{m} \hat{\alpha}_i B_{i,n}(x)$$

where $B_{i,n}(x)$ Bernstein basis.

we estimate $\hat{\alpha}_i$ with local polynomial by minimizing the locally weighted sum of squares. we control the size of neighborhood around $\frac{i}{n}$ by the bandwidth. Using the minimizing criterion (generalized cross validation), we may identify optimal parameters, such as bandwidth and degree of Bernstein basis.To demonstrate the efficacy of the approach suggested in this research, examples of simulated data are provided.

Key Words: *Local polynomial regression, Bandwidth, Bernstein basis, Polar forms, Quasi-interpolation.*

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pseudo almost periodic solution of a Delayed Quaternion-Valued Fuzzy Recurrent Neural Networks model on Time Scales

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Abstract:

we consider quaternion-valued fuzzy recurrent neural networks with time-varying delays on time scales. Different from the previous literature, we use a direct method to obtain our theoretical results to avoid decomposing the model into real-valued or complex valued systems. Then, we obtain some sufficient conditions on the existence, uniqueness, and Sp-global exponential stability of weighted Stepanov-like pseudo almost periodic solution on time scales of the considered model by applying inequality analysis techniques on time scales, a fixed point theorem, and composition theorem, and by constructing an appropriate Lyapunov function. At the end of this work, we give a numerical example and simulations to illustrate the effectiveness of the obtained results.

Key Words: Time scales, Quaternion-valued recurrent neural networks, Fuzzy theory,Weighted Stepanov-like pseudo almost periodicity, global stability.

Introduction: As we know, discrete and continuous recurrent neural networks play a key role in theoretical research and applications. Also, discrete-time neural networks are more beneficial and convenient for numerical simulation and computation than continuous-time NNs. Hence, not only do we need to study continuous-time neural networks, but we also need to study discrete NNs. To avoid the difficulties of studying the dynamic properties of continuous and discrete systems, respectively, it is helpful to study these properties on time scales, which Stefan Hilger ([1]) introduced in his PhD thesis in order to unify continuous and discrete analysis. As a result, using time scale dynamic systems, subjects such as the existence of a solution, stability, floquet theory, periodicity, and the dynamics of NNs can be studied more precisely and broadly. Recently, the existence and stability of the periodic solution on time scales has been one of the most attractive themes in the context of various kinds of abstract dynamic equations , partial dynamic equations ([2]), integro-dynamic equations ([3]) and general dynamic systems ([4]).
Motivated by the above statement, we summarized the innovation points of this paper as follows : (I) we integrate fuzzy operations into quaternion-valued RNNs with time-varying delays on time scales. (II) For the time being, this is the first time that theWSpPAP dynamics of a delayed QVFRNNs are being investigated on time scales, which can unify both continuous time and discrete time cases of RNNs. The QVFRNNs proposed in this work also contain real-VFRNNs and complex- VFCNNs as their special cases. (III) We take into account another oscillation space that has never been taken into account in the different classes of recurrent neural networks.

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Numerical Methods for Solving the One-Dimensional Birkhoff Polynomial Interpolation Problem

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Abstract:

The subject of this Presentation aims to study the interpolation Birkhoff polynomial [3]. Given a set $Z = \{z_0, ..., z_n\}$ with $(n+1)$ nodes, of ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$), we will seek to study questions of existence and of uniqueness [7, 4, 2, 6] of a polynomial P such that P and a number of its derivatives take, at these nodes, given values, as well as its representation [1, 5]. The algorithmic aspect will also be taken into consideration, and a numerical implementation of the obtained solutions must be carried out. Complexity studies algorithmic, numerical stability and comparisons with other solutions are expected from this work.

Key Words: *Birkhoff interpolation - lacunar polynomial - Schur complement - Recursive algorithm*

Introduction: *In 2019, Messaoudi [5], invented recursive methods to solve Lagrange and Hermite interpolation problems, he used the notion of Schur's complement, to obtain recursive methods to accelerate convergence.*

Unlike the interpolation of Lagrange and Hermite, the Birkhoff interpolation problem is very difficult for the following reasons

- *the solvency of the problem : (conditional solvency).*
- *suppose we know the solvency. It is difficult to obtain an explicit representation of the solution.*

Although we can obtain an explicit representation in some special cases.

In 1955, Turán pointed out the fundamental questions of Birkhoff interpolation are

1. *(Existence problem)*
2. *(Uniqueness problem)*
3. *If solution exists and is unique, how can it be represented in the most appropriate form? (Representation problem)*
4. *Questions related to convergence.*

In this Presentation, we propose new algorithms to solve the problem of Birkhoff polynomial interpolation [1], these new algorithms will be called :

- *Birkhoff Recursive Polynomial Interpolation Algorithm (BIRPIA): This algorithm determines the interpolation space in a recursive manner.*

- *Recursive Lacunar Polynomial Interpolation Algorithm (RLPIA): The interpolation space set at the start. It is based on a matrix writing of the problem and exploits the properties of the Schur complement to solve systems of linear equations.*

Some of its properties will be studied and some examples will be also given. This Presentation is organized as follows. In section 1, we recall the History of the problem posed, some of properties of existence and unicity. In section 2, we will present a theoretical resolution. In section 3, we show how to build the new algorithm of the first approach to numerical resolution BIRPIA and prove some of its properties. In section 4, we show how to build the new algorithm of the second approach to numerical resolution RAPIA and prove some of its properties, we also give a gneralisation of the RAPIA called RLPIA. Sections 5, 6 and 7 are concerned with comparisons between the solutions obtained, the study of the complexity algorithmic and numerical stability.

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The Finite Element Method in Anisotropic Sobolev Spaces

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Abstract:

In this work we study the various aspects of the theory and implementation of finite element methods for elliptic boundary value problems in anisotropic sobolev spaces,we present some basic results on anisotropic Sobolev spaces and we obtain a weak formulation, we show the existence and uniqueness of the solution of the variational formulation by the Lax-Milgram theorem.By the finite element method we obtain optimal error estimates for the Galerkin approximation in anisotropic sobolev spaces.

Key Words: *Finite element method, Anisotropic Sobolev spaces, Elliptic boundary values problem*

Introduction: *We consider elliptic boundary value problems of the form*

$$\begin{aligned} Au &= f, \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \Gamma, \end{aligned}$$

where A is the partial differential operator defined by

$$Au = \sum_{i=1}^{N} D_{x_i} \left(a_i(x) D_{x_i} u \right)$$

and f is given.Here $D_{x_i}u$ *denote* $\frac{\partial u}{\partial x_i}$ *. We assume that the coefficient* $a_i(x)$ *is sufficiently smooth and that*

$$0 < \underline{a} \leq a_i(x) \leq \bar{a}, \quad \text{for } x \in \bar{\Omega}$$

where Ω *is a bounded open subset of* $\mathbb{R}^N (N \geq 2)$ *with smooth boundary,* $1 < p_1 ¡ p_2 < \ldots < p_N < +\infty$ *be a N real numbers and* $\vec{p} = (p_1, \ldots, p_N)$*.where the anisotropic Sobolev space*

$$W^{1,\vec{p}}(\Omega) = \left\{ u \in L^{p_N}(\Omega), \frac{\partial u}{\partial x_i} \in L^{p_i}(\Omega), i = 1, 2, \ldots, N \right\}$$

and its dual $W^{-1,\vec{p}'}(\Omega)$ *where* $\overrightarrow{p'} = (p'_1, \ldots, p'_N)$ *with* $p'_i = \frac{p_i}{p_i - 1}$.

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Partial Differential Equations

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Stabilization for a class of unbounded bilinear control system in Banach space

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Abstract:

This work aims to study the problem of stabilization for the following infinite dimensional bilinear system

$$\begin{cases} \dfrac{dx(t)}{dt} = Ax(t) + \eta(t)BCx(t), \quad t \geq 0 \\ x(0) = x_0 \end{cases} \tag{1}$$

Here, A is the infinitesimal generator of a linear C^0-semigroup on a Banach space X with domain $D(A)$. The linear operator B is bounded from a Banach space U to the extrapolation space X_{-1} of X. The linear operator C is bounded from a Banach space Z to U . The real-valued function $\eta(\cdot)$ denotes the control.
Using a regularizing approach, we prove the existence and uniqueness of the systemâs solution (1). Under sufficient conditions, we prove the stability of the solution by using an adequate bounded feedback control law. Application to the transport equation is considered.

Key Words: *Unbounded bilinear control system, Stabilization*

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Double Phases Problems : Insight and new trends

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Abstract:

The primary goal of this paper is to identify multiple solutions for singular elliptic problems associated with a quasilinear obstacle equation, governed by the double phase obstacle operator. Utilizing Ricceri's variational principle, Bonanno's three critical points theorem, and several appropriate assumptions, we demonstrate the existence of at least one solution and two distinct nontrivial solutions under broad conditions on the nonlinearity.

Key Words: *Recceri Theorem - Three xeak solution- Variational methods*

Introduction: .....

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Quasilinear degenerate elliptic unilateral problems with measure data in the Anisotropic Sobolev Space

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Abstract:

we consider a nonlinear elliptic unilateral equation whose model is

$$-\sum_{i=1}^{N}\partial^{i}\sigma_{i}(x,u,\nabla u)+L(x,u,\nabla u)+N(x,u,\nabla u)=\mu-\operatorname{div}\phi(u)\quad\text{ in }\Omega. \tag{1}$$

We prove the existence of entropy solutions for (1) in the anisotropic Sobolev space, under the hypotheses, $\mu = f - \operatorname{div} F$ belongs to $L^1(\Omega) + W^{-1,p'}(\Omega)$, The nonlinear terms $L(x, s, \nabla u)$ satisfy the sign and growth conditions, $N(x, s, \nabla u)$ verifies only the growth conditions.

Key Words: *Anisotropic Sobolev spaces, Entropy solutions, nonlinear elliptic equation*

Introduction: *study of anisotropic problems is one of the topics from the field of calculus of variations and partial differential equations, indeed there are a large number of researches that have introduced anisotropic Sobolev spaces which are the appropriate framework to study non-linear problems involving a growth exponent $\vec{p}$, where $\vec{p}$-laplacien is the differential operator formulated by*

$$\Delta_{\vec{p}}(u)=\sum_{i=1}^{i=N}\partial_{x_i}\left(\left|\partial_{x_i}u\right|^{p_i-2}\partial_{x_i}u\right),$$

which generalize the p -laplace operator.
In the Lebesgue-Sobolev spaces, Benilan introduced the approach of entropy solutions adjusted to Boltzmann .
L. Boccardo and their co-authors have been treated the problem (1) *with*

$$Au=-\sum_{i=1}^{N}\frac{\partial}{\partial x_i}\left(\left|\frac{\partial u}{\partial x_i}\right|^{p_i-2}\frac{\partial u}{\partial x_i}\right),$$

and f is a bounded Radon measure on Ω.
Bendahmane et al. proved an existence solutions of the problem (1) *in the case where*

$$Au=-\sum_{i=1}^{N}\frac{\partial}{\partial x_i}\sigma_i\left(x,\frac{\partial u}{\partial x_i}\right),$$

and where the data $f=(f_1,\ldots,f_m)^{\top}$ is vector-valued Radon measure on Ω.
Benboubker et al. have been shown existence results for some anisotropic elliptic problem likes

$$-\operatorname{div}a(x,u,\nabla u)+N(x,u,\nabla u)+|u|^{p_0(x)-2}u=f-\operatorname{div}\phi(u).$$

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Recently the mathematical researches dealing with existence of solutions to some problems parabolic and elliptic under a different assumptions is massive.
The main difficulty in this study comes from the fact that the classical coercivity condition not assumed on the operator Au in the space $W^{1,\vec{p}}(\Omega)$. To beat this difficulty, we will use the term $\frac{1}{n}|u|^{p_0-2}u$ said (penalization term) in the approximates equation in order to show our desired theorem.
We introduce the assumptions on $\sigma_i(x,s,\xi)$, for which the problem (1) *admits a solution. In the final section we present the proof of our main results.*

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Study of Nonlinear Elliptic Equations with Measurable Boundary Conditions in Anisotropic Weighted Sobolev Spaces

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Abstract:

The novelty of this work lies in establishing the existence of entropy solutions for anisotropic elliptic equation

$$-\operatorname{div} B(x,\nu,\nabla\nu) + H(x,\nu) = f - \operatorname{div} F$$

where the datum $f \in L^1(\Omega)$ and $F \in \prod_{i=1}^{N} L^{p_i'}(\Omega, w_i^*)$ and $H(x,\nu) \in L^1(\Omega)$. Furthermore, only the large monotonicity conditions will be assumed on $B(x,s,\xi)$. To overcome this difficulty we will use the approach of Minty's lemma in the anisotropic weighted Sobolev spaces.

Key Words: *Anisotropic weighted Sobolev spaces, Nonlinear elliptic equations, Entropy solutions, Minty's lemma, Monotonicity conditions*

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Nonlinear Parabolic Problem with L^1-data in Orlicz Spaces

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Abstract:

In this study, we discuss the solvability of a nonlinear parabolic problem modeled by

$$\begin{cases} \dfrac{\partial b(u)}{\partial t} - \operatorname{div}(a(x,t,u,\nabla u) + \Phi(x,t,u)) = f & \text{in } Q_T, \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ b(u)(t=0) = b(u_0) & \text{in } \Omega; \end{cases}$$

where $A(u) = -\operatorname{div}(a(x,t,u,\nabla u))$ is a Leray-Lions type operator defined on the inhomogeneous Orlicz–Sobolev space $W_0^{1,x}L_M(Q_T)$, and $\Phi(x,t,u)$ is a nonlinear lower-order term.

The function $b : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function in $C^1(\mathbb{R})$, and the second term f belongs to $L^1(Q_T)$.

Key Words: *Nonlinear parabolic problems, Orlicz spaces, Renormalized solutions, Existence.*

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A nonlinear elliptic problems with natural growth and integrable data .

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In this paper, we study the existence of a renormalized solution to the problem of the nonlinear degenerate elliptic equation

$$-div\,(a(x,u,Du)) + H(x,u)|Du|^2 = f,$$

where the first term is a Lerray-Lions operator and blow-up for a finit value m of the unknown u, $H(x,u)$ is nonlinearity satisfying the growth conditions and without sign conditions, and $f \in L^1(\Omega)$.

Key Words: *Renormalized solutions, Blow-up, $L^1 - data$.*

Introduction: .....

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The Nehari manifold for anisotropic Kirchhoff problems involving variable singular exponent and critical terms

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Abstract:

In this paper, we investigate multiplicity results for the following anisotropic Kirchhoff-type problems with a singular term and a critical nonlinearity that changes sign

$$\begin{cases} -\sum_{i=1}^{N} M\left(\int_\Omega |\partial_i u|^{p_i} dx\right) \partial_i \left(|\partial_i u|^{p_i-2}\partial_i u\right) = \frac{f_1(x)}{u^{\beta(x)}} + \lambda f_2(x) u^{\bar{p}^*-1} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded regular domain in $\mathbb{R}^N$, $N > \bar{p}$ and the critical Sobolev exponent, denoted as $\bar{p}^*$, is defined as $\bar{p}^* = N\bar{p}/(N-\bar{p})$, where $\bar{p} = N/\sum_{i=1}^{N} \frac{1}{p_i}$. Additionally, we have the parameter $\lambda > 0$ and the exponent variable $0 < \beta(x) < 1$. Here, f_1 is a positive function, while f_2 is a function that changes sign, and M models a Kirchhoff coefficient. We establish the existence of at least two weak solutions that have different energy sign. Our approach relies on the fibering method in the form of the Nehari manifold.

Key Words: *Anisotropic Kirchhoff-type problems, Nehari manifolds, fibering method, critical nonlinearities, singular problems, exponent variable.*

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Entropy solution for some parabolic problems nonlinear in Musielak spaces with L^1 Data

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Abstract

In this study , we deal with the existence result for nonlinear parabolic equations of the form: $\begin{cases} \frac{\partial u}{\partial t} + Au - div\phi(x,t,u) = f & \text{in } \Omega_T \\ u(x,0) = u_0(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega X(0,T) \end{cases}$

where $A(u) = -div(a(x,t,u,\nabla u)$ is a Leary-Lions type operator defined on the inhomogeneous Orlicz–Sobolev space $W_0^{1,x}L_\varphi(\mathbf{\Omega}_T)$, φ is a Musielak function, $\Phi(x,t,u)$ is a carathéodory function , the second term $f\ in\ L^1(\mathbf{\Omega}_T)$. We estabilsh the existence result of entropy solution in the setting of Musielak-Orlicz spaces.

Keywords: Nonlinear parabolic problems, Musielak-Orlicz spaces, Entropy solution, Existance.

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On a generalized $p(x)$-biharmonic problem with two nonlocal terms

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Abstract:

Using variational methods, we obtain in the present work, the multiplicity of nontrivial weak solutions for a class of generalized $p(x)$-biharmonic problem involving two nonlocal terms and two real parameters λ, μ with indefinite weight under no flux boundary condition. The novelty here can be seen as a generalization and continuation to some existing results.

Key Words: *Variable exponent, Generalized p(x)-biharmonic problem, Variational methods, No-flux boundary condition.*

Introduction: *Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with a smooth boundary and $N \geq 2$.*
This work deals mainly with the following problem which is subject to the no-flux boundary condition

$$(P_{\lambda,\mu}) \begin{cases} K(\Psi(u))\big(b(x,u)\big) + \lambda c(x,u) = d(x,u) + \mu h(x,u)\big[\int_\Omega H(x,u)dx\big]^r & \text{in } \Omega, \\ u = constant,\ \Delta u = 0 & \text{on } \partial\Omega, \\ \int_{\partial\Omega} \frac{\partial}{\partial n}[a(x,\Delta u)\Delta u]ds = 0, \end{cases}$$

where

$$\begin{aligned} b(x,u) &= \Delta(a(x,|\Delta u|)\Delta u) + e(x)a(x,|u|)u; \\ c(x,u) &= |u|^{s(x)-2}u; \\ d(x,u) &= n(x)|u|^{q(x)-2}u, \end{aligned}$$

$\lambda \in \mathbb{R}, \mu$, r are a positives parameters, $n \in L^{\beta(\cdot)}(\Omega)$ is a changing sign weight, K is a function fulfilling certain conditions. $q(\cdot)$, $s(\cdot)$ and $\beta(\cdot) \in C_+(\overline{\Omega}) = \left\{ l \in C(\overline{\Omega}) : l^- = \min_{x\in\overline{\Omega}} l(x) > 1 \right\}$.
The functional Ψ is defined by

$$\Psi(u) = \int_\Omega \left[A(x,|\Delta u|) + e(x)A(x,|u|)\right] dx.$$

with $A(x,y) = \int_0^y a(x,\zeta)\zeta d\zeta$.
We denote by $C^{0,\frac{1}{|log t|}}(\overline{\Omega})$ the set of all function $l : \overline{\Omega} \to \mathbb{R}$ that are log-Hölder continuous, that is, there exists $\overline{e} > 0$ such that,

$$\text{for all } x,y \in \overline{\Omega} \text{ with } 0 < |x-y| \leq \frac{1}{2}, \quad \text{we have} \quad |l(x) - l(y)| \leq \frac{\overline{e}}{-log|x-y|}.$$

In this work, we assume that $p(\cdot) \in C_+(\overline{\Omega}) \cap C^{0,\frac{1}{|log t|}}(\overline{\Omega})$.

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Renormalized solution for a fractional (s,p)-Laplacian parabolic problem with diffuse measure data

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Abstract:

We prove existence of *renormalized solution* for a fractional (s,p)-Laplacian parabolic problem whose model is

$$(\mathcal{P})\begin{cases} u_t + (-\Delta)_p^s u(x) = \mu \text{ in } Q := (0,T)\times\Omega, \\ u(0,x) = u_0(x) \text{ in } \Omega,\ u(t,x) = 0 \text{ on } \partial Q := (0,T)\times\partial\Omega, \end{cases}$$

where $(-\Delta)_p^s u$ is the fractional (s,p)-Laplace operator (with $ps < N$, $0 < s < 1$ and $p > 2 - \frac{s}{N}$), $u_0 \in L^1(\Omega)$ and $\mu \in \mathcal{M}(Q)$ (the vector space of all finite Radon measures in Q). We first prove some a priori estimates for the fractional parabolic p-capacity then we discuss the main properties of solutions without using the decomposition of the right-hand side.

Key Words: *Fractional order Sobolev spaces; Capacity, Fractional Laplacian, Dirichlet boundary conditions, Existence; Renormalized solutions*

Introduction: *Let $\Omega \subset \mathbb{R}^N$ be an open bounded domain, $T > 0$, $p > 2 - \frac{s}{N}$ (with $ps < N$ and $0 < s < 1$), and let us consider the evolution problem*

$$(\mathcal{P})\begin{cases} u_t + (-\Delta)_p^s u(x) = \mu \text{ in } Q := (0,T)\times\Omega, \\ u(0,x) = u_0(x) \text{ in } \Omega,\ u(t,x) = 0 \text{ on } \partial Q := (0,T)\times\partial\Omega \end{cases} \tag{1}$$

where $u \mapsto (-\Delta)_p^s u$ is the so-called fractional p-Laplace operator, which, up to renormalization factors, is defined as

$$\begin{aligned} (-\Delta)_p^s u(t,x) &:= \mathbf{P.V.} \int_{\mathbb{R}^N} \frac{|u(t,x)-u(t,y)|^{p-2}(u(t,x)-u(t,y))}{|x-y|^{N+ps}} dy \\ &= \lim_{\epsilon\downarrow 0} \int_{\mathbb{R}^N\setminus B_\epsilon(x)} \frac{|u(t,x)-u(t,y)|^{p-2}(u(t,x)-u(t,y))}{|x-y|^{N+ps}} dy, \end{aligned} \tag{2}$$

where $(t,x) \in \mathbb{R}^N \times \mathbb{R}^+$ and $\mathbf{P.V.}$ is commonly used abbreviation for "in the principal value sense", u_0 is a function in $L^1(\Omega)$ and $\mu \in \mathcal{M}(Q)$ is any measure with bounded variation over Q. The purpose of this study is to give different properties of the fractional capacity in connection with Radon measures under which the existence result holds for renormalized solutions, namely, we characterize the fractional order Sobolev spaces for which the fractional capacity is defined and we show existence and regularity results for generalized solutions under suitable assumptions on the data. In our study, we will be only concerned with nonlocal equations in the case $p \neq 2$ which makes the existence result more difficult since the operator turns out to be nonlinear and parabolic. However, we stress that, even when $p = 2$, while sufficiently regular data, the existence result for duality solutions is not trivial. We aim at establishing rather general existence results of renormalized solutions when μ is diffuse. To this purpose, we use alternatively either the approach of [7, 10] (see also [1, 2, 6, 8]), based on a key result, namely the equidiffuse property related to a different type of approximation coupled with a truncation property of μ mostly relying on its

nonlinear potential. We will also exploit one more idea consisting in the characterization of the solution in terms truncation, this method relies on the fundamental work [3] used for conservation laws and the one of the existing formulations in the elliptic case (see [4, 5]) which suggests the use of an adequate truncation problem to establish some capacitary estimates on the level sets of the solution, and to derive a subsequence to obtain a limit function which is a renormalized solution by virtue of the convergence results.

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On a class of double phase problem involving potentials terms

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Abstract:

In this study, we show that there exists a non-negative, non-trivial solution to a class of double phase problems involving potentials allowing for vanishing behavior at infnity, in the context of Sobolev-Orlicz spaces with variable exponents in complete compact Riemannian n-manifolds. The Nehari manifold and other variational methods are used in our approach.

Key Words: Existence of solutions. Double phase operator. Laplacian. Variational problems. Sobolev spaces with variable exponents on Complete manifolds

Introduction: .....

References

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NON-COERCIVE DIFFERENTIAL INCLUSIONS PROBLEMS IN ANISOTROPIC SOBOLEV SPACES WITH VARIABLE EXPONENT AND L^1-DATA

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Abstract:

The focus of this work lies in investigating the existence of solutions for a general class of nonlinear elliptic problems characterized by anisotropic variable exponents, coupled with differential inclusion, degenerate coercivity and L^1-data. Our study establishes the existence of entropy solutions for this class of non-coercive differential inclusions. Additionally, we will provide insights into the regularity properties of these solutions.

Key Words: *Anisotropic Sobolev spaces with variable exponents, Entropy solutions, Maximal monotone graph, Degenerate coercivity.*

Introduction: *Our aim is to study existence and regularity of entropy solutions for nonlinear elliptic problems of the form:*

$$(B,f)\ \begin{cases} -\sum_{i=1}^{N} D_i\left(\frac{A_i\left(x,D_iu\right)}{\left(1+|u|\right)^{\lambda(p_i(x)-1)}}\right)+\gamma(u)\ni f \ \text{ in } \ \Omega, \\ u=0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N (N\geq 2)$, $\partial\Omega$ its Lipschitz boundary, $f\in L^1(\Omega)$, $p_i:\overline{\Omega}\longrightarrow(1,\infty)$, $i=1,...,N$ are continuous functions, $\gamma:\mathbb{R}\to 2^{\mathbb{R}}$ is a set valued, maximal monotone mapping such that $0\in\gamma(0)$, $D_iu=\dfrac{\partial u}{\partial x_i}$ denotes the partial derivative of u with respect to x_i and $A_i:\Omega\times\mathbb{R}\to\mathbb{R}$, $i=1,...,N$, are Carathéodory functions satisfying the classical Leray-Lions assumptions [7].
In the problem (B,f), the difficulties are due to: Firstly, the degeneracy of the monotone operator

$$\Lambda(u):=-\sum_{i=1}^{N} D_i\left(\frac{A_i\left(x,D_iu\right)}{\left(1+|u|\right)^{\lambda(p_i(x)-1)}}\right)$$

when $0<\lambda<1$, which can result in a slow diffusion effect when the solution u becomes large. As a result of this degeneracy, classical methods for elliptic equations are not applicable, even when the datum f is regular.
Secondly, when an anisotropic variable exponential growth condition is given for $A_i(\cdot,\cdot)$ in the equation. In this case, the operator Λ exhibits a more complex nonlinearity, making some techniques used in the constant exponent case inapplicable.
Thirdly, the nonlinearity of $\gamma(u)$.
Motivated by [1, 3, 2], let us emphasize that the main task in this work is to extend the results in [2] to anisotropic variable exponent case [4, 5, 6]. We will study the existence and some $q_i(x)$-regularity properties of entropy solutions for the nonlinear multivalued elliptic problem (B,f) with L^1 data, considering any maximal monotone graph γ where the functional framework is anisotropic Sobolev spaces with variable exponents.

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On a class of $p(z)$-biharmonic Kirchhoff type problems with indefinite weight and no-flow boundary condition

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Abstract:

In this paper we study the existence of weak solutions for a fourth order variable exponent Kirchhoff type problem involving $p(z)$-biharmonic operator with indefinite weight and no flux boundary condition. The proof of the existence result relies on employing the concept of a Fredholm-type results for a pair of nonlinear operators $(\mathfrak{O}, \mathfrak{S})$, in conjunction with the theory of variable exponent Sobolev spaces.

Key Words: *Weak solution, No-fux boundary condition, KirchhofF type problem, Variable exponent, $p(z)$-biharmonic operator, Fredholm-type.*

Introduction: *examination of operators of this kind is of considerable interest in various disciplines, including electrorheological fluids (as discussed in [4, 8]), steady-state thermorheological viscous flows of non-Newtonian fluids, elastic mechanics (as explored in [10]), image processing (refer to [2]), and the mathematical modeling of phenomena like gas filtration through porous media (refer to [1]).*
Problem $(\mathcal{P}_\Theta^\lambda)$ concerns the stationary variant of the Kirchhoff equation initially proposed by Kirchhoff in 1883 [5]. This equation extends the classical d'Alembert wave equation by incorporating changes in string length during vibrations. The introduction of the function $\mathcal{M}$ in problem $(\mathcal{P}_\Theta^\lambda)$ transforms it into a nonlocal problem, where the equations are no longer pointwise. This characteristic introduces mathematical intricacies that make the investigation of such problems particularly captivating.

Talbi et al. have studied in [9] Kirchhof type problem :

$$(\mathcal{P}_\lambda)\begin{cases} M(L(\vartheta))\left(\Delta^2_{p(z)}\vartheta + a(z)|\vartheta|^{p(z)-2}\vartheta\right) = \lambda f(z,\vartheta) & \text{in } \mathcal{D}, \\ \vartheta = \ const,\ \Delta\vartheta = 0 & \text{on } \partial\mathcal{D}, \\ \displaystyle\int_{\partial\mathcal{D}} \frac{\partial}{\partial n}\left(|\Delta\vartheta|^{p(z)-2}\Delta\vartheta\right) ds = 0, \end{cases}$$

they demonstrated the existence and multiplicity of nontrivial weak solutions for a fourth order variable exponent Kirchhof-type problem involving $p(x)$-biharmonic operator. This was achieved by applying the Mountain Pass theorem along with the theory of variable exponent Sobolev spaces. For other related results, we refer to [3, 7].

Inspired by the referenced literature, we examine problem $(\mathcal{P}_\Theta^\lambda)$ under more general conditions than those considered in [9]. Within this expanded framework, we demonstrate the existence of a weak solution for problem $(\mathcal{P}_\Theta^\lambda)$ by utilizing topological results akin to those discussed by Dinca in [6] of a Fredholm-type nature.

5th edition of International Conference on Applied Mathematics (ICAM'2024), December 19 - 20, 2024, Taza, Morocco.

The objective of this study is to establish the existence of weak solutions for a Kirchhoff-type problem denoted as $(\mathcal{P}_\Theta^\lambda)$*, defined as follows:*

$$(\mathcal{P}_\Theta^\lambda)\begin{cases}\mathcal{M}(\mathcal{U}(\vartheta))(\Delta^2_{p(z)}+\Theta_1(z)|\vartheta|^{p(z)-2}\vartheta)+\Theta_2(z)|\vartheta|^{\alpha(z)-2}\vartheta \\ \qquad\qquad = \lambda_1\mathcal{L}_1(z,\vartheta,\nabla\vartheta)+\mathcal{L}_2(z,\vartheta)|\vartheta|^{\eta_2(z)}_{\eta_1(z)} & \text{in } \mathcal{D},\\ \vartheta = \ const,\ \Delta\vartheta = 0 & \text{on } \partial\mathcal{D},\\ \displaystyle\int_{\partial\mathcal{D}}\frac{\partial}{\partial n}\left(|\Delta\vartheta|^{p(z)-2}\Delta\vartheta\right)d\eta = 0.\end{cases}$$

Here, $\mathcal{D}\subset\mathbb{R}^N$ *(with* $N\geq 2$*) is a bounded domain with a smooth boundary* $\partial\mathcal{D}$*.*
The exponents $p(z),\alpha(z)\in C^+(\Omega)$*,whith* $2\leq\alpha^-\leq\alpha(x)\leq\alpha^+<p^-\leq p(x)\leq p^+<\infty$*, where* $p(z)$ *is a log-Hölder continuous function (to be specified in Section 2). The parameters* $\Theta_1,\Theta_2\in L^\infty(\Omega)$*, and the functions* $\mathcal{L}_2:\Omega\times\mathbb{R}\to\mathbb{R}$ *and* $\mathcal{L}_1:\Omega\times\mathbb{R}\times\mathbb{R}^N\to\mathbb{R}$ *are Carathéodory functions satisfying the assumption of growth.*

The remainder of the article is organized as follows. In Section 2, we review the definitions of variable exponent Lebesgue spaces $L^{p(z)}(\mathcal{D})$ *and Sobolev spaces* $W^{k,p(z)}(\mathcal{D})$*, highlighting essential properties relevant for our analysis. Section 3 is dedicated to presenting and proving the main results of this study.*

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ON SOME NONLINEAR ELLIPTIC EQUATIONS WITH MEASURABLE BOUNDARY CONDITIONS IN ANISOTROPIC WEIGHTED SOBOLEV SPACES

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Abstract:

Abstract. The novelty of this note is to establish existence result for the following anisotropic elliptic equation

$$-\operatorname{div} B(x,\vartheta,\nabla\vartheta) + H(x,\vartheta) = f - \operatorname{div} F$$

where the datum $f \in L^1(\Omega)$ and $F \in \prod_{i=1}^{N} L^{p_i'}(\Omega, w_i^*)$ and $H(x,\vartheta) \in L^1(\Omega)$. Furthermore only the large monotonicity conditions will be assumed on $B(x,s,\xi)$. To overcome this difficulty we will use the approach of Minty's lemma in the anisotropic weighted Sobolev spaces.

Key Words: *Anisotropic, weighted Sobolev spaces, elliptic problem, Entropy solutions, Measure data,Truncation.*

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Existence of a solution for nonlinear degenerate elliptic unilateral problems

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Abstract:

The present work deals on the existence of a solution for a nonlinear unilateral problem of the form: $-div(\nu_1 a(x,u,\nabla u))+\Theta(x,u,\nabla u)\nu_2 = f$ in the framework of the weighted Sobolev space $W_0^{1,p}(\Omega,\nu_1,\nu_2)$, where ν_1,ν_2 are two $\mathcal{A}_p$-weights and $f \in L^1(\Omega)$. Although $\Theta(x,\tau,\zeta)$ is a nonlinear term having a growth condition with respect to ζ and non-growth concerning τ, but it fulfills a sign condition on τ.

Key Words: *Weighted Sobolev spaces; Nonlinear degenerate elliptic equations; Class of $\mathcal{A}_p-$ weights; Unilateral problem.*

Introduction:

Let Ω be a bounded smooth domain in $\mathbb{R}^N$ ($N \geq 2$) and ν_1,ν_2 two weight functions belonging to the $\mathcal{A}_p$-weight class. Let $A(u) = -div(\nu_1 a(x,u,\nabla u))$ be a Leray-Lions operator acting from the weighted Sobolev space $W_0^{1,p}(\Omega,\nu_1,\nu_2)$ into its dual $W^{-1,p'}(\Omega,\nu_1,\nu_2)$ $\left(p' = \frac{p}{p-1}, 1 < p < \infty\right)$.
The purpose of this work is to show the existence of a solution for the nonlinear degenerate elliptic unilateral problem :

$$\begin{cases} -div(\nu_1 a(x,u,\nabla u)) + \Theta(x,u,\nabla u)\nu_2 = f \text{ in } D'(\Omega), \\ u \in W_0^{1,p}(\Omega,\nu_1,\nu_2), \Theta(x,u,\nabla u) \in L^1(\Omega,\nu_2), \end{cases}$$

where Θ is a nonlinear degenerate lower-order term including some natural growth with respect to $\mid \nabla u \mid$. Regarding u, we suppose that there are no growth restrictions, however it satisfies the sign condition, and the data f is in $L^1(\Omega)$.
Our objective is to involve sufficient conditions on a,Θ ensuring the existence of at least one solution for the nonlinear degenerate elliptic problem with Dirichlet boundary condition:

$$\begin{cases} u \in K_\phi, \Theta(x,u,\nabla u) \in L^1(\Omega,\nu_2), \\ \langle A(u), T_k(v-u)\rangle + \displaystyle\int_\Omega \nu_2 \Theta(x,u,\nabla u) T_k(v-u)\ \mathrm{d}x \geq \langle f, T_k(v-u)\rangle \\ \text{for all } v \in K_\phi \text{ and all } k > 0, \end{cases} \tag{1}$$

where ϕ is a measurable function, $K_\phi = \left\{v \in W_0^{1,p}(\Omega,\nu_1,\nu_2), v \geq \phi \text{ a.e. in } \Omega\right\}$.

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Internal and gradient stabilization of an unbounded bilinear systems: parabolic and hyperbolic cases

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Abstract:

This study considers internal and gradient feedback stabilization of an unbounded bilinear system in parabolic and hyperbolic cases. It provides sufficient conditions for internal and gradient weak and strong stabilization.

Key Words: *Bilinear system, Internal stabilization, gradient stabilization, unbounded operator, nonlinear semigroup*

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Bilinear stabilization of second order evolution equations by a class of bilinear unbounded feedbacks

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Abstract:

In this paper, we consider the problem of feedback stabilization of the following bilinear unbounded systems:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} - \alpha(u)\, \frac{\partial u}{\partial t}(\xi, t)\, \delta_\xi = 0, \ (x,t) \in (0,1) \times (0,+\infty), \\ u(0,t) = u(1,t) = 0, \quad t \in (0,+\infty), \\ u(x,0) = u^0(x), \ \frac{\partial u}{\partial t}(x,0) = u^1(x), \ x \in (0,1), \end{cases} \tag{1}$$

where $\xi \in (0,1)$ and δ_ξ is the Dirac mass concentrated in the point $\xi \in (0,1)$.
In this case, the uniform stability is not guaranteed. Thus, we have provided an explicit weak decay rate for all regular initial data using the weak observability properties for the undamped problem and the boundness of the transfer function. A similar frameworks are considered in ([1], [2],[3],[4],[5],[6]).

Key Words: *Bilinear Stabilization, Unbounded control operator.*

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Existence of mild solution and approximate controllability for nonlinear fractional neutral evolution systems

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Abstract:

In the present study, we investigate the existence of mild solutions and approximate controllability for the following Riemann–Liouville fractional neutral evolution systems with nonlocal conditions of order $0 < q < 1$.

$$\begin{cases} D^q_{RL}\left[z(t) - h\left(t, z(t)\right)\right] = -\mathcal{A}z(t) + \mathcal{B}u(t) + f(t, z(t)), & t \in J = (0, T], \quad (T \geq 1), \\ I_t^{1-q}\left[z(t) - h(t, z(t)\right]|_{t=0} = z_0. \end{cases}$$

The Laplace transform and semigroup theory are the tools used to prove the existence. Furthermore, the approximate controllability of such systems is proved on the basis of the Nemytskii operator, the Mittag–Leffler function and certain hypotheses using fixed point theorems as well as the construction of a Cauchy sequence. An example is provided to highlight the main results.

Key Words: *Fractional Neutral Evolution System, Approximate Controllablity, Mild Solution, Existence of solutions, Riemann–Liouville Derivatives.*

Introduction: *Fractional differential equations are a generalization of classical ones, they have been included in modeling a variety of biological, physical, mechanical, and engineering problems, etc. [1, 2, 3, 4]. A wide range of practical applications has prompted the publication of several existing results incorporating the two classical derivatives Caputo and Riemann–Liouville [5, 7] for the systems with order* $0 < q < 1$.
Neutral differential equations are those where the time delay appears under the derivative of the unknown functions, many control systems are governed by neutral differential equations type. That is why dealing with such systems is more complex than the classical ones, where the delays only occur in the state. This type of delays can be complex to handle but it improves the performance of the system where they occur. Adding neutral delays to a fractional system is more beneficial as such system does have memory.
Neutral systems can simulate a wide range of natural phenomena from many fields such as fluid dynamics, electronics, biological models, chemical dynamics, etc. see [9, 10]. Classical differential equations cannot describe most of these phenomena, such as heat conduction in a fading memory material, and anomalous diffusion.

Controllability is a very important concept in contemporary science and technology. Control is ubiquitous in our daily lives, it exists in our cars as we drive and maintain the car on the road, in the braking system, and so on.

Broadly speaking, the concept of exact controllability of a system means transferring the system from an initial state to a final state within a fixed time interval, although this is sometimes difficult to achieve; there is a weaker concept, that of approximate controllability, where systems have a weaker conceptualization and a wider application. In general, it is difficult to achieve exact controllability for differential systems in infinite-dimensional Banach spaces, and many diffusion control systems are not exactly controllable since the corresponding linear operator generates a compact semi-group, for example, the heat equation. For instance, we refer the readers to the references [11, 12]. As a consequence, many researchers are devoted to the study of approximate controllability because it is more realistic and adequate in many real situations [14, 13]. Nevertheless, the theory of approximate controllability for fractional equations is still in its early stages.

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Stochastic Analysis and Probability Theory

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CONVERGENCE AND DECOMPOSITION THEOREMS OF SET-VALUED MILS

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Abstract:

We explore the properties and convergence theorems of set-valued martingale in the limit (mil), extending classical results from the theory of martingales to the set-valued context. Specifically, we focus on establishing convergence theorems for convex closed-valued mil under various conditions. Furthermore, we provide quasi-decomposition theorems for set-valued mil. Finally, we investigate the optional stopping theorem for set-valued mil, examining under what criteria it can be garanted.

Key Words: *Random sets; Set-valued martingales; Set-valued mils; Decompositions.*

Introduction: *Set-valued stochastic processes, which extend the classical notion of stochastic processes to sets, provide a robust framework for modeling uncertainty in scenarios where outcomes are represented as sets of possible values rather than single points. Due to the application of these processes in fields like economics, optimisation, statistics and image analysis, its theoretical exploration has garnered significant attention, with one notable area being set-valued martingales, which represent fair games or processes where future values are expected to be equal to the current value given past information .*

Over the years, several important theoretical results have been established in the study of set-valued martingales. For example, authors such as F.Akhiat and al. [1] , C.Hess [4], F.Hiai and H.Umegaki [5], and N.Papageorgiou [6] have contributed convergence theorems for set-valued martingales, submartingales, and supermartingales under different conditions. N.Papageorgiou [6] also provided the optional sampling theorem for set-valued martingales.

By the development of stopping time techniques, it is allowed the generalization of martingale concept. This advancement has led to the introduction of vector-valued mil by M.Talagrand [8] and set-valued mil by C.Castaing and F.Ezzaki [2]. This progress led to significant results, such as the convergence theorems for convex weakly compact-valued mils by C.Castaing and F.Ezzaki [2], and K.Tahri [7].

However, several questions remain open, such as whether similar convergence results can be achieved for convex closed-valued mils, and whether the optional stopping and the optional sampling theorems can be garanted. Additionally, an important question is whether a set-valued mil can be decomposed like vector-valued mil.

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Simulated annealing estimation for three parameters in a generalized stochastic Raleigh diffusion process

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Abstract:

Modeling real data using stochastic processes is a very useful and powerful tools for many fields of research, such as science and technology, infectious diseases, biology and others. In this context, we introduce a new stochastic diffusion process related the generalized Rayleigh distribution (see, [3]). Then, we identify the analytic solution of the stochastic differential equation, the transition probability density function and the mean functions (see, [2, 6]). To estimate the three process parameters in this process, we use the maximum likelihood method based on discrete sampling (see, [1, 4]). The system of likelihood equations has no explicit solution, so the form of the maximum likelihood estimators cannot be obtained. It is essential to use numerical methods such as the simulated annealing algorithm (see, [5, 7]). We will finish by investigating the possibility of using this new process to fit and predict simulated data.

Key Words: *Generalized Raleigh distribution; Stochastic diffusion process; Maximum likelihood estimation; Simulated annealing; Statistical inference; fit data.*

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Convergence of martingale : Application to uniform amart

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Abstract:

The convergence of martingales with values in RNP Banach space is well known in the literature. In this paper, we extend this result to multivalued martingales as well as uniform amart, providing a broader framework for convergence in such contexts.

Keywords: *Multifunction, martingales, uniform amart*

Introduction: *The convergence of martingales and uniform amarts is a central topic in probability theory, particularly in the framework of Banach spaces with the Radon-Nikodym property (RNP).*
These spaces provide a foundation for addressing a wide range of problems, including almost sure convergence and the study of asymptotic behavior in stochastic processes.
However, few results exist concerning the study of multivalued stochastic processes.
In this paper, we aim to generalize the results already established for a vector valued martingale to a martingale with values in subset of E. As application we establish the convergence of multivalued uniform amarts.

Notations and Definitions

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space, $(\mathcal{F}_n)_{n\geq 1}$ an increasing sequence of sub σ-algebras of $\mathcal{F}$ such that $\mathcal{F}$ is the σ-algebra generated by $\cup_{n\geq 1}\mathcal{F}_n$, E is a separable Banach space with the dual E^*.
Let $cc(E)$ a set of closed and convex subsets of E.
A multivalued function X is $\mathcal{F}$-measurable if $X^-(O) = \{\omega \in \Omega : X(\omega) \cap O \neq \emptyset\} \in \mathcal{F}$, for every open set $O \subset E$.
X is integrable if $\int_\Omega d(0, X)d\mathbf{P} < \infty$.
A sequence $(X_n)_{n\geq 1}$ of measurable multivalued functions with values in $cc(E)$ is said to be adapted to $(\mathcal{F}_n)_{n\geq 1}$ if for any $n \geq 1$, X_n is $\mathcal{F}_n$-measurable.

Definition 1 *An integrable adapted sequence $(X_n, \mathcal{F}_n)_{n\geq 1}$ is said to be a martingale if $X_n = E^{\mathcal{F}_n}X_{n+1}$ a.s. for each $n \geq 1$.*

Definition 2 *An integrable adapted sequence of multivalued functions $(X_n, \mathcal{F}_n)_{n\geq 1}$ is said to be a multivalued martingale if $X_n = E^{\mathcal{F}_n}X_{n+1}$ a.s. for each $n \geq 1$.*
Where $E^{\mathcal{F}_n}X_{n+1}$ is the conditional expectation of X_{n+1} with respect to $\mathcal{F}_n$.

Definition 3 *An integrable adapted sequence $(X_n, \mathcal{F}_n)_{n\geq 1}$ is said to be uniform amart if*

$$\lim_{\tau\in T}\sup_{\sigma\geq\tau} E\|E^{\mathcal{F}_\tau}X_\sigma - X_\tau\| = 0,$$

where σ and τ are bounded stopping times.

Definition 4 *An integrable adapted sequence of multivalued functions $(X_n, \mathcal{F}_n)_{n\geq 1}$ is said to be uniform amart if*

$$\lim_{\tau\in T}\sup_{\sigma\geq\tau} EH(E^{\mathcal{F}_\tau}X_\sigma, X_\tau) = 0,$$

where H is the Hausdorff distance.

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Exploring Complex Monge-Ampère Measures : Insights from Projective Logarithmic Potentials

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Abstract

We study the projective logarithmic potential $\mathbb{G}_\mu$ of a probability measure μ on the complex projective space $\mathbb{P}^n$. We prove that the range of the operator $\to \mathbb{G}_\mu$ is contained in the (local) domain of definition of the complex Monge–Ampère operator acting on the class of quasi-plurisubharmonic functions on $\mathbb{P}^n$ with respect to the Fubini– Study metric. Moreover, when the measure μ has no atom, we show that the complex Monge–Ampère measure of its logarithmic potential is an absolutely continuous measure with respect to the Fubini–Study volume form on $\mathbb{P}^n$.

Keywords : Projective logarithmic potential, complex Monge–Ampère measure.

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Accurate Closed-Form Approximations for the Sum of a Ratio of Uncorrelated Rayleigh Random Variables

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Abstract:

In this work, we propose an accurate approximate method for the distribution of the sum of a ratio of uncorrelated Rayleigh random variables (URRVs). In particular, we derive closed-form expressions for the probability density function(PDF), moment generating function(MGF), and cumulative distribution function(CDF) in terms of Univariate Fox H-Function(UFHF). To this end, the accuracy of our approach is validated through Monte Carlo Simulations for several scenarios.

Key Words: *Cumulative distribution function, moment generating function, probability density function, rayleigh distribution, sum-ratio of random variates, univariate Fox H-function.*

Introduction: *approximating key statistical functions such as probability density function (PDF), cumulative distribution function (CDF), and moment generating function (MGF) of the sum-ratio of random variables is fundamentally essential in various fields. In wireless communications, they represent a very useful tool to investigate the statistical behavior of a system performance metrics, such as signal-to-noise ratios (SNRs), which often depend on the ratios of random variables. In addition, the sum of these ratios commonly occurs in scenarios with multiple antennas, cooperative communication strategies, and fading channels, where transmitted signals face various forms of interference and noise. Over the years, Rayleigh random variables (RRVs) have been employed to model the fading amplitude in wireless communication scenarios with rich scattering, where signals are subject to multiple reflections and diffractions due to the absence of a direct line of sight (LOS) between transmitter and receivers [1] . However, deriving exact expressions for various critical performance metrics remains a significant challenge due to the presence of complex integrals that can not be solved analytically. As a result, there has been an increasing interest in proposing accurate approximations for the PDF, CDF and MGF of various distributions. In [2], an infinite series representation is used to obtain the CDF of the sum of RRVs precisely. A relatively simple and widely used small argument approximation (SAA) method is proposed in [3] to get a simple closed-form approximation to the Rayleigh sum PDF by modifying the SAA so that it is accurate for a wide range of arguments. In [4], an approximation to the PDF for the product of n arbitrary independent RRVs is proposed, based on a transformed Nakagami- m distribution. In [5]- [7], authors proposed accurate approximations for the distribution of the product, ratio and sum of Weibull random variables. More recently, closed-form approximations for the PDF and the CDF of the sum of correlated Weibull random variables and the sum-product of Rayleigh random variables are derived in terms of the univariate Meijerâs G-function (UMGF) in [8] and [9], respectively. The parameters of the UMGF are estimated using the moment-based method by evaluating the first four moments of each distribution. In this paper, we propose an approximation approach for the distribution of the sum-ratio (SR) of N Ã N uncorrelated rayleigh random variables (URRVs) using the UFHF instead of the UMGF used previousely. To this end, closed-form expressions for the PDF, CDF and MGF of the SR of URRVs are derived and validated through Monte Carlo simulations.*

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Global Existence of Solutions of Stochastic Aggregation Equations with Random Diffusion.

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Abstract:

In Stochastic aggregation equations like the Patlak Keller Segel model, the issue of global existence versus finite time blow-up is addressed in this work. Finite time blow-up is typically the result of solutions to these equations being known to exist only locally in time in the absence of diffusion. The main question is whether global existence can be restored by introducing stochastic noise, specifically random diffusion.

Key Words: *Aggregation diffusion equation, keller Segel equation, Random diffusion, Globale solution*

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Stochastic Aggregation-Diffusion Equation: Analysis via Dirichlet Forms

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Abstract:

In this article, we study the stochastic aggregation-diffusion equation with a singular drift represented by a monotone radial kernel. We demonstrate the existence and uniqueness of a diffusion process that acts as a weak solution to our equation. This process can be described as a distorted Brownian motion originating from a delocalized point. Utilizing Dirichlet form theory, we prove the existence of a weak solution for a quasi-everywhere point in a state space. While uniqueness is not assured for solutions commencing from points outside polar sets, explicitly characterizing these sets poses a significant challenge. To address this, we employ the H_2-condition introduced by [1]. This condition provides a more thorough understanding of the uniqueness issue within the framework of Dirichlet forms. Consequently the H_2-condition is pivotal in enhancing the analysis of weak solutions, which ensures a more detailed comprehension of the problem. An explicit expression for the generalized Schrodinger operator associated with certain kernels is also provided.

Key Words: *Stochastic aggregation-diffusion equation, Diffusion process, Dirichlet form, Distorted Brownian motion*

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